NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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XXXIII - EXPERIMENTAL DETERMINATION OF THERMAL AND HYDRODYNAMICAL BEHAVIOR OF AIR FLOWING ALONG FINNED PLATES

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SUMMARY

An experimental investigation has been performed on the thermal and hydrodynamical behavior of air flowing along a steam-heated plate to which were attached cylindrical fins 5/16 inch in diameter and 5/8 inch long. Local values of unit thermal conductance per row of fins were obtained for air flowing in a "variable-width" rectangular duct (variation of hydraulic diameter). In this duct were placed for the various tests a plate having metallic fins, one having nonmetallic fins, and also a plate without fins. The range of Reynolds modulus was from 67,800 to 418,000, based on the hydraulic diameter of the duct. The results obtained with all three plates are compared and the effects of "finning" and of the added turbulence along the unfinned portions of the plate due to the presence of the fins are shown. The effect of duct width on heat transfer is also given. The results of temperature and pressure traverses across the duct section normal to the main-stream flow are presented for one duct width and one weight rate. A method of predicting the performance of a pin-finned plate is presented and compared with experimental results.

Although the system presented here consisted of cylindrical pin fins the method of analysis may be extended to systems with other types of fins.

INTRODUCTION

Very few data on cylindrical or pin fins are available for the design of heat exchangers. At present it is necessary for the designer to postulate a composite system based on separate data for flow over tube banks, or cylinders, and a flat plate (reference 1). This procedure neglects one of the most important items, namely, the added heat transfer due to the turbulence induced by the fins on the air adjacent to the unfinned portion of the plate. Consequently, data concerning the effect of pin fins upon the heat-transfer characteristics of a flat plate were obtained. Because this effect is composed of two parts, (1) the effect of an extended surface and (2) the turbulence-promoting effect of the rows of fins, the tests were carried out with steel fins which would produce both effects, with wooden fins which would produce only the second effect, and with an unfinned "flat" plate which would produce neither effect. By comparing the performance of the steel-pin plate with that of the unfinned flat plate, the over-all effect of "finning" can be determined. Comparison of the performance of the wooden-pin plate with that of the unfinned flat plate shows the effect, on the unfinned portion of the plate, of the turbulence induced by fins because very little heat (about 10 percent) is transferred by the wooden fins. The effect of thermal conductivity in the fins is obtained by comparison of the performance of the steel-pin plate with that of the wooden-pin plate.

The numerical data obtained are to be used only for the specific system studied here. The object of the investigation, however, was a generalized study of an extended surface on a finned heat-transfer system such as is encountered in exhaust-gas and air heat exchangers. Thus the method of analysis used is probably applicable to systems other than that of pin fins.

The following data were obtained:

(1) Weight rates of air through duct containing test plates

(2) Temperature of air at inlet and outlet of test section

(3) Heat-transfer rate (by measurement of volume rate of condensation of steam for specified sections of test plates)

(4) Over-all static-pressure drops over test plate for various separations of the duct wall from the plate

(5) Velocity and temperature traverses upstream from, and at several points along, the plate for one duct width and weight rate

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DESCRIPTION OF APPARATUS

The test plates (fig. 1) consisted of rectangular sections of flat steel $16\frac{3}{4}$ by $15\frac{7}{16}$ by $\frac{1}{8}$ inches. Cylindrical fins were fastened to two such plates, the fins of one plate being constructed of steel and those of the second plate being of wood. A third plate which had no fins fastened to it (unfinned) was also tested. (See photographs of plates and test equipment in figs. 2 to 4.) One thermocouple was placed in the inlet duct upstream from the plate to measure the air temperature; two more were placed on the plate to measure the plate temperature; and another was mounted in the duct downstream from the plate to measure the outlet-air temperature. The thermocouples were fixed in one position and were constructed of 26-gage iron-constantan wires.

The backs of the test plates were provided with eight condensate-collecting sections as shown in figures 1, 4, and 5. These sections were constructed of thin sheet iron and were soldered to the plate. The plate assembly was fitted into a rectangular chest in which there was steam to cover the back of the plate. A 750-watt superheater placed in the entering steam line was used to superheat the steam a few degrees in order to eliminate carry-over of entrained liquid into the steam chest and especially into the condensate-collecting sections. The outside of the steam chest was lagged with 85-percent-magnesia brick. Eight 25-cubic-centimeter thermally insulated burettes, located beneath the plate, were used to meter the condensation rate of the steam in the sections. Copper tubes at the bottom of each section passed through the plate and guided the condensate through rubber connectors into the burettes.

The heat-transfer section was placed in one side of a rectangular, wooden duct (fig. 6). The interior of the duct was 77 inches long and 12 inches high. One side of the duct was movable, allowing the width to be varied from 5/8 inch (against end of pins) to $5\frac{1}{4}$ inches. Pressure taps were placed at positions $12\frac{1}{2}$ inches upstream and $20\frac{3}{4}$ inches downstream from the test plates. Two calibrated, sharp-edged, flat-plate orifices were used in the air-metering section. A description of the metering section, the duct system, and the blower which supplied the air can be found in reference 2.

METHOD OF ANALYSIS

Experimental Heat-Transfer and Pressure-Drop Data

Heat transfer.- From the data obtained, the effective thermal conductance (per vertical row of 12 fins) is determined as a function of the distance from the leading edge of the heated test plate, the duct width, and the weight rate of air through the duct for the three test plates (unfinned flat plate, wooden-fin plate, and steel-fin plate).

The heat transferred per row of fins from the condensing steam in the steam chest to the air flowing in the duct q_r is calculated in the following manner:

$$q_r = \frac{\Delta h_{vap}(R_{av} - R'_{av})}{i} \tag{1}$$

where Δh_{vap} is the heat of vaporization for water at atmospheric pressure, R_{av} is the average rate of condensation per section under "load" conditions, R'_{av} is the average rate of condensation under "no load" conditions, and i is the number of vertical rows of pins per condensing section. The terms "load" and "no load" apply to the test conditions in which air was passed and not passed, respectively, over the test plate while condensation data were taken. (For convenience, a summary of the symbols used in this report is given in appendix A.)

The mixed-mean temperature increase of the air for each vertical row is obtained by making a heat balance for the air. The rate of heat transfer from the steam as calculated in the preceding paragraph must be equal to q, the heat gained by the air stream, or

$$iq_r = q = Wc_p \, \Delta\tau \tag{2}$$

where iq_r is the heat loss of the steam for a certain condensing section of the test plate, W is the weight rate of air, c_p is the heat capacity of the air, and $\Delta\tau$ is the increase in temperature of the air over the length of this section. The mixed-mean temperature of the air at each of the rows of the test plates τ_r, based on the inlet-air temperature τ_o, must be

$$\tau_r = \tau_o + \Delta\tau \tag{3}$$

or

$$\tau_r = \tau_o + \frac{\frac{q_r}{2} + \sum_{0}^{N-1} q_r}{Wc_p} \tag{4}$$

The effective thermal conductance per row $(fA)_e$ is then calculated from the equation

$$q_r = (fA)_e(t_p - \tau_r)$$

or

$$(fA)_e = \frac{q_r}{(t_p - \tau_r)} \tag{5}$$

where t_p, the plate temperature, is obtained experimentally. This equation is the defining expression for the effective thermal conductance.

Static-pressure drop.- Because the pressure taps were situated $12\frac{1}{2}$ inches upstream and $20\frac{3}{4}$ inches downstream from the test section, the measured over-all static-pressure drop is that of the test section plus 33 inches of unfinned duct. For finned test sections, the pressure drop in 33 inches of unfinned duct was calculated to be less than 10 percent of the over-all pressure drop. In figures 7 and 8 the static-pressure-drop data calculated for only the test section are presented.

Prediction of Heat Transfer and Static-Pressure Drop for Unfinned Rectangular Duct

Heat transfer.- The ideal system approached in the experiments on the unfinned flat plate is that in which a turbulent velocity distribution prevails at the inlet to the test section and the temperature distribution is uniform at the inlet. Thus the velocity gradient at the surface $\left(\frac{du}{dy}\right)_{y=0}$ and the static-pressure drop are constant along the test section. The temperature gradient at the wall and the heat transfer, however, decrease from infinity (theoretically) at the inlet to some finite value downstream.

The effective thermal conductance as a function of x, the distance from the inlet, or leading edge, along the test section, is approximated by the following equations (reference 1):

For

$$0 < \frac{x}{D_H} < 4.4 \times 2 = 8.8$$

(see section entitled "Unfinned plates" under DATA TAKEN AND CORRELATION WITH ACCEPTED EQUATIONS)

$$f_{c_x} = 7.3 \times 10^{-4} \; T_f^{0.3} \frac{G^{0.8}}{x^{0.2}} \tag{6}$$

or

$$(fA)_{e_x} = 7.3 \times 10^{-4} \; T_f^{0.3} \frac{G^{0.8}}{x^{0.2}} A_x \tag{6a}$$

and for

$$\frac{x}{D_H} > 4.4 \times 2 = 8.8$$

$$f_{c_x} = 5.4 \times 10^{-4} \; T^{0.3} \frac{G^{0.8}}{D_H^{0.2}} \tag{7}$$

or

$$(fA)_{e_x} = 5.4 \times 10^{-4} \; T^{0.3} \frac{G^{0.8}}{D_H^{0.2}} A_x \tag{7a}$$

where

D_H hydraulic diameter, feet

f_{c_x} local unit thermal conductance, Btu/(hr)(sq ft)(°F)

T_f one-half sum of plate absolute temperature and mixed-mean air absolute temperature, °R

G weight rate of fluid per unit area, (lb)/(hr)(sq ft)

$(fA)_{e_x}$ local effective thermal conductance, Btu/(hr)(°F)

A_x local heat-transfer area, square feet

T mixed-mean air absolute temperature, °R

Although equation (6) is derived for another system, that in which both the velocity and temperature distributions are uniform at the inlet, reference 3 shows that the equation is representative of both systems.

The average effective thermal conductance $(fA)_{e_{av}}$ over the length of the test section is obtained by integrating equations (6a) and (7a):

For

$$0 < \frac{l}{D_H} < 4.4 \times 2$$

$$(fA)_{e_{av}} = \frac{A}{l} \int_0^l f_{c_x}\,dx = 9.1 \times 10^{-4}\, T_f^{0.3} \frac{G^{0.8}}{l^{0.2}} A \tag{8}$$

and for

$$\frac{l}{D_H} < 4 \times 4$$

$$(fA)_{e_{av}} = \frac{A}{l} \int_0^l f_{c_x}\,dx = 5.4 \times 10^{-4} \frac{T^{0.3} G^{0.8}}{D_H^{0.2}} \left(1 + 1.1 \frac{D_H}{l}\right) A \tag{9}$$

where l is the over-all heated length along the test section from the leading edge, in feet, and A is the heat-transfer area.

Pressure drop.- The static-pressure drop is calculated from the equation for a straight duct (reference 1):

$$\frac{(5.2\ \Delta p)}{\gamma} = \zeta\ \frac{l_s}{D_H}\left(\frac{u_m^2}{2g}\right) \tag{10}$$

where

Δp	static-pressure drop, inches of water
γ	weight density of the fluid, pounds per cubic foot
ζ	friction factor (reference 4)
l_s	length of test section, feet
u_m	mean fluid velocity, feet per second
g	gravitational force per unit mass, (lb)/(lb)(sec^2/ft)

Prediction of Heat Transfer and Static-Pressure Drop

for Rectangular Pin-Finned Duct

Heat transfer.- The prediction of the heat transfer or effective thermal conductance and the static-pressure drop for the several experimental conditions was made by postulating composite systems in which available information on fluid flow and heat transfer over tube banks, individual pin fins, and flat plates was used.

For the case where there is no heat loss from the ends of the fins (insulated duct wall against ends of fins; that is, $y_o = 5/8$ in.) the effective thermal conductance per vertical row of fins on the test plate is calculated from the following equation (reference 1):

$$(fA)_{e_x} = n\sqrt{f_{F_x}PkA_F}\ \tanh\left(\sqrt{\frac{f_{F_x}PL^2}{kA_F}}\right) + f_{u_x}A_{u_x} \tag{11}$$

where

n	number of pins per vertical row
f_{F_x}	point unit thermal conductance along fin, Btu/(hr)(sq ft)($^{\circ}$F)

P perimeter of fin measured on plane parallel to base of fin, feet

k thermal conductivity of fin material, Btu/(hr)(sq ft)(°F/ft)

A_F cross-sectional area of fin perpendicular to direction of heat flow in fin, square feet

L length of fin projecting into stream, feet

f_{u_x} point unit thermal conductance along unfinned surface, Btu/(hr)(sq ft)(°F)

A_{u_x} area of flat plate not covered by fins, square feet

For the case where there is heat loss from the ends of the fins, the effective thermal conductance per vertical row of pin fins on the finned plates is calculated from the following equation (reference 4, p. II-32):

$$(fA)_{e_x} = n\sqrt{f_{F_x} P k A_F}\left(\frac{\sinh m_x L + \frac{f_E}{k m_x}\cosh m_x L}{\cosh m_x L + \frac{f_E}{k m_x}\sinh m_x L}\right) + A_u f_{u_x} \qquad (12)$$

where $m_x = \sqrt{f_{F_x}P/(kA_F)}$, f_E is the unit thermal conductance over the ends of the fins, and A_u is the unfinned surface area of the test section.

The three unit thermal conductances for air appearing in equations (11) and (12) are obtained from the following equations:

$$f_{u_x} = 7.3 \times 10^{-4}\, T_f^{0.3} \frac{G^{0.8}}{x^{0.2}} \quad \text{for} \quad 0 < \frac{x}{D_H} < 4.4 \times 2 \qquad (6)$$

$$f_{u_x} = 5.4 \times 10^{-4}\, T^{0.3} \frac{G^{0.8}}{D_H^{0.2}} \quad \text{for} \quad \frac{x}{D_H} > 4.4 \times 2 \qquad (7)$$

$$f_E = 9.1 \times 10^{-4}\, T_f^{0.3} \frac{G^{0.8}}{l_E^{0.2}} \qquad (13)$$

$$f_{F_x} = 14.5 \times 10^{-4}\, F_{a_x} T_f^{0.43} \frac{G^{0.6}}{d^{0.4}} \qquad (14)$$

where

l_E equivalent length of end of fin, equal to $d/\sqrt{2}$, feet

F_{a_x} point value of "tube-arrangement modulus" for tube banks (see DISCUSSION)

d diameter of fin, feet

The use of equations (6), (7), (13), and (14) is based on the following postulates:

(1) The unfinned portion of the plate is equivalent to a flat plate (equation (6)) or a duct (equation (7)) depending upon the distance from the entrance to the test section

(2) The ends of the fins are equivalent to flat plates of length l_E, defined after equation (14)

(3) The flow of air around the fins is similar to the flow of air over a tube bank

These postulates form the ideal system which approaches the actual system.

Pressure drop.- For the case in which the pin fins extend across the width of the duct, forming a tube bank (y_o = 5/8 in.), the static-pressure drop is calculated by the method of references 5, 6, and 7. (See fig. 7(a).)

$$\Delta p = \frac{KNG^2}{10.84 \times 10^8 \gamma} \tag{15}$$

where

$$K = \frac{\left(0.044 + \dfrac{0.08b}{(a - 1)^{0.43+(1.13/b)}}\right)}{(Re_d)^{0.15}}$$

is a friction factor presented graphically as a function of Re_d, a, and b in references 5, 6, and 7 (K being represented by f in these references) and where

N	rows of pin fins in direction of air flow
b	longitudinal pitch, diameters
a	transverse pitch, diameters

DISCUSSION

The heat-transfer investigation presented in this report was performed on three different test plates to ascertain the effects of finning a flat plate, namely, the effect of turbulence on the heat-transfer rate along the unfinned portions of the plate and the effect of extended surfaces. The effect of turbulence generated by fins was obtained by comparing the heat transferred from a wooden-finned plate (without an analytical estimate of the heat transferred by the wooden fins) with that from an unfinned plate. The effect of heat conducted in the extended surfaces (pin fins) was obtained by comparing the heat transferred from a steel-finned plate to that from a wooden-finned plate.

An attempt also was made to correlate the experimental data with predictions based on equations representative of two fundamental flow systems for unfinned test sections. These two systems may be described briefly as:

(1) Flow over flat plates: The thermal and hydrodynamic boundary layers are postulated to originate at the same point and these boundary layers have not filled the duct.

(2) Flow inside ducts: The velocity and temperature distributions are postulated to be fully established before the inlet to the test section.

The system studied in the course of this investigation was a combination of these two, because a thermal boundary layer was initiated at the inlet to the test section but the hydrodynamic boundary layer was already fully established at this point. The tests for unfinned ducts showed that system (2) probably applied to a duct width of 5/8 inch. The value of $L/D_H < 4.4$ was used in former reports (references 1 and 3) to distinguish between systems (1) and (2) for flow in straight ducts with heat transfer occurring around the entire periphery. However, for the tests reported here, the thermal boundary layer is initiated from only one side of a rectangular duct so that the distinguishing value of L/D_H may be 8.8. This value is derived upon considering the insulated boundary of the duct as the center line of an equivalent duct of diameter equal to twice the distance between the insulated and uninsulated boundaries of the test section.

The data tend to indicate that the principal criterion for the application of the equation for system (1) (flow over flat plates) is the initiation of a thermal boundary layer.

It must be noted here that steady-state heat losses are inherent in the type of apparatus used in this investigation. These "no-load" heat losses were evaluated by taking data with no air flowing through the duct. Free-convection heat losses from the test plate were minimized by placing asbestos packing in the duct. This procedure was justified because the free-convection heat losses from the test plate were estimated to be less than 1 percent (reference 1) of the total heat transferred under load conditions. Evidence that these losses were evaluated correctly is given by the accuracy of the heat-balance ratios, which were within 20 percent of unity in most cases. Repeat runs showed that the data taken on the test apparatus were reproducible to about 10 percent.

The data for the first point $(x = 1.3 \text{ in.})$ should be used for qualitative analysis only. Quantitative accuracy of the first point was sacrificed to permit greater versatility of the equipment. The same fact holds true for the last point, but in this case the effect is almost negligible, since the magnitude of the losses is small compared with the quantity measured. Many of the results presented in graphical form were obtained by interpolation and extrapolation of the original data which are given in tables I, II, and III and in figures 9 to 11. The extrapolated data are indicated on the curves.

DATA TAKEN AND CORRELATION WITH ACCEPTED EQUATIONS

Variation of Local Effective Thermal Conductance with Distance

Figures 9 to 11 show the local effective thermal conductance per row of fins for each of the three test plates as a function of the distance from the leading edge of the heated test plate with the weight rate of air per unit cross-sectional area as a parameter. The parameter was based, in all cases, on the average weight rate and the minimum area normal to flow (that is, the free area at a plane through the center lines of the fins in one vertical row). For purposes of comparison, values of the effective thermal conductance per row of fins were used for all test plates, including the unfinned plate. The unit thermal conductance f_{u_x} at any point of the unfinned plate is equal to 12 times the value of the effective thermal conductance $(fA)_{e_x}$ because the area which one row of fins occupies is 1/12 square foot.

Unfinned plates.- The variation of the effective thermal conductance with distance from the leading edge for the unfinned plate is approximated by equations (6) and (7), in their respective regions, to within 10 percent (fig. 11). The fact that equation (6) (flow over flat plates) approximates the experimental data indicates that the initiation of a thermal boundary layer alone is sufficient to permit application of this equation. Of course, for the case in which $x/D_H > 8.8$ (approximately), it is thought that the inlet effects have become negligible, so that the equation for flow in ducts is applicable. The L/D_H ratio for all duct widths with the exception of 5/8 inch is less than 8.8 and so is considered to be a flat-plate system. The L/D_H ratio for the 5/8-inch duct width is 13, so that for most of its length this system should also be considered as a flat plate. (See comparison in fig. 11(a).)

Finned plates.- In the case of the finned-plate systems, the prediction of the effective thermal conductance $(fA)_e$ is made for the downstream region where the inlet effect is negligible. These predicted values are presented in table IV. Analytic calculations were made to predict:

(a) Heat transferred through wooden fins only

(b) Heat transferred through steel fins only

(c) Heat transferred along unfinned plate (considered to be a flat plate for all duct widths except $y_o = 5/8$ in. when the unfinned plate was considered a wall of a duct)

(d) Heat transfer for unfinned area $(fA)_u$ (same as item (c) with correction for heat-transfer area occupied by fins)

(e) Heat transfer from wooden-finned plate (items (a) plus (d))

(f) Heat transfer from steel-finned plate (items (b) plus (d))

Corresponding measured values for items (c), (e), and (f) are shown for comparison.

When the movable duct wall was placed against the ends of the pin fins $(y_o = 5/8 \text{ in.})$ and there was consequently no heat loss from the ends of the pin fins, equation (11) is used for prediction of the heat transfer. Because of the similarity of the system to a narrow tube bank, the prediction of the local unit thermal conductance over the pins f_{F_x} is made by using equation (14), which is the equation for predicting the heat transferred by air flowing in tube banks. Equation (14) is based on the use of the maximum weight rate per unit area (minimum flow area). By the use of equation (11) (no heat loss from end of fin) the experimental values of $(fA)_e$ for the steel-finned plate were predicted to about 15 percent for the 5/8-inch duct width. The values for the wooden-finned plate were predicted to about 35 percent for the 5/8-inch duct width but to about 15 percent for larger duct widths.

When the ends of the fins are exposed to the flow of air $(y_o > 5/8 \text{ in.})$, heat is transferred from the ends of the fins. The prediction is based, in this case, on equation (12) and the prediction of the local unit thermal conductance over the fins again is made with equation (14). Because the movable duct wall does not touch the ends of the fins, some of the air bypasses the finned section and consequently determination of the proper value of G to use in the prediction is difficult. If the average weight rate per unit cross-sectional area G is used in equation (14) to predict f_{F_x}, and this value of f_{F_x} is used in equation (12) for prediction of $(fA)_e$, then the predicted values of $(fA)_e$ agree well with the mean of the values experimentally obtained at each duct width. However, because the heat loss from the ends of the pin fins is less than 10 percent (by calculation) of the total heat transfer, equation (11) approximates the experimental data almost as well as equation (12).

The bypassing of part of the air produces an irregular velocity distribution which varies with x and y_o. The variations for $y_o = 3\frac{1}{8}$ inches are shown in figure 12, where velocity and temperature traverses plotted against x are presented. Use of these velocity distributions was made in the prediction of $(fA)_e$.

The values of G, the weight rate per unit cross-sectional area in equations (13) and (14), were taken from figure 13 to be 0.7 (to evaluate end loss) and 0.5 (to evaluate f_c over fins of the average weight rate per unit area). With this assumption, $(fA)_e$ was calculated for $y_o = 3\frac{1}{8}$ inches and G = 5000 pounds per hour per square foot. These calculations are shown in the following table:

System	Effective thermal conductance, $(fA)_e$ (Btu/(hr)(°F)(row))		
	Measured values	Predicted values	
		With average G	With corrected G
Steel-finned plate	1.1	1.44	0.95
Wooden-finned plate	.6	.51	.33
Unfinned plate	.36	.37	.22
Steel fins only		1.10	.75
Wooden fins only		.17	.13
Unfinned area $(fA)_u$		.34	.20

These data should be analyzed in the light of recent investigations concerning the effect of roughness on velocity distribution and heat-transfer rates. (See reference 8.)

Further investigation of the variation of the local value of the tube-arrangement modulus in the tube banks must be made. This tube-arrangement modulus is used to take into account the higher unit thermal conductance attained over a row of tubes which is located in the wake of a similar row of tubes. Reference 1 gives the average value of F_a for tube banks containing one or more rows of tubes. For tube banks with 10 or more rows of tubes, the average (and local) value of F_a is 1.43.

Variation of Average Effective Thermal Conductance with Duct Width

Figure 14 reveals the average effective thermal conductances for flow of air over the steel-finned plate, wooden-finned plate, and unfinned flat plate, as a function of the duct width, with the weight rate per unit area as a parameter. The average effective thermal conductance of the steel-finned plate decreases slightly as the duct width increases, whereas that of the wooden-finned plate decreases rapidly to a minimum value at a duct width of about 1 inch, then increases with increasing duct widths. This minimum might be ascribed to the bypassing of the air in the space between the ends of the fins and the duct wall (see velocity distribution, fig. 12) because of the lower resistance to flow in this space. Since the geometry and the flow conditions were the same for both the steel-finned plate and the wooden-finned plate the phenomenon must also occur for both test plates. The lack of a minimum in the steel-finned plate may be due to the simultaneous occurrence of a counterbalancing mechanism. Such a mechanism might be the additional heat transfer from the ends of the steel fins, which would be negligible in the wooden fins when the ends of the fins are uncovered. However, the foregoing explanation is not satisfactory because the minimum is quite pronounced and the results obtained by a comparison of equations (11) and (12) show that in this case the heat transfer from the ends of the fins should be a minor effect. Further analysis is required in order to explain these data.

Variation of Average Effective Thermal Conductance with Weight Rate per Unit Cross-Sectional Area

The variation of $(fA)_{e_{av}}$ for the unfinned flat plate as a function of the weight rate per unit area is presented in figure 15. The data fall along a line whose slope is 0.82, which agrees closely with the slope of 0.80 required by equations (8) and (9). Equation (8), for flow over flat plates, is plotted to show the agreement. The data for $(fA)_{e_{av}}$ at all duct widths fall on the same line to within about 15 percent of the values given by equation (8).

The fact that the average values of $(fA)_e$ fall on the same curve, no matter what the duct width, and the fact that the equation for flow over flat plates predicts the experimental data rather closely indicate that, as stated previously, the principal criterion for application of the equation for flow over a flat plate is the initiation of a thermal boundary layer.

EFFECTS OF FINNING

The over-all effect of finning as a function of x, the distance from the initiation of the thermal boundary layer, is presented in figure 16. This figure shows the ratio of the effective thermal conductance of the steel-finned plate to that of the unfinned flat plate plotted against x. The weight rate per unit cross-sectional area is used as a parameter. In order to study the individual effects of finning, the over-all effect was separated into two parts: That due to turbulence induced by the presence of the fins and the change in velocity distribution between the fins and that due to extended heat-transfer surfaces of the pin fins.

Effect of turbulence.- The effect of added turbulence along the unfinned portions of the plate due to the presence of the fins is shown in figure 17, where the "corrected" ratio of the local effective thermal conductance of the wooden-fin plate to that of the flat plate is plotted against distance. Reasonably reliable local values of this ratio for the $5\frac{1}{4}$-inch plate spacing were as high as 1.75, which indicates an increased rate of heat transfer as high as 75 percent due to the turbulence caused by the pin fins. (It must be remembered that for the tests employing the wooden-fin and unfinned test plates with large duct widths, the heat-transfer rates were small. Hence, a slight error in the unfinned flat plate would magnify the error in the aforementioned ratio. Also, data from the steel-finned plate were not used to determine the "turbulence effect" because the heat transfer through the unfinned portion of the plate was a smaller fraction of the total rate.) The average effect of turbulence (over the entire plate) is given in figure 18(a) as a function of the duct width and the weight rate per unit cross-sectional area. It can be seen that at some duct widths this effect is negligible. Figure 18(b) shows the variation of the turbulence effect with weight rate per unit flow area for $y_o = 5/8$ inch (tube bank), which yields the largest effect of all duct widths tested.

It is seen that the ratios of the local and of the average effective thermal conductances decrease and approach unity with increasing weight rate per unit area. This variation indicates that as the weight rate per unit area was increased, the heat transfer through the unfinned area increased more rapidly than that through the fins. By consideration of the application of equations (7), (14), and (12), this phenomenon could be deduced analytically.

The heat rate along the unfinned plate area varies as the 0.8 power of G while the heat rate through the pins varies approximately as the 0.4 power so that the heat rate along the plate becomes the more important term at large values of G.

Further tests should be made with a staggered arrangement of fins because the added turbulence is of greater magnitude for this case.

The minimum point in figure 18(a), where the effect of turbulence is shown as a function of duct width and weight rate per unit flow area, is low because of the unexplained data for the wooden-fin plate. (See section entitled "Finned plates" under DATA TAKEN AND CORRELATION WITH ACCEPTED EQUATIONS.)

Actually, the change in the heat transfer ascribed to the flow characteristics is not a function of the added turbulence alone. Accompanying the increase in turbulence, a change occurs in the velocity distribution in the air stream. These two effects should be determined separately in future investigations.

Effect of extended surfaces.- The effect of added heat-transfer area by means of finning is shown in figure 19 where the ratio of the local effective thermal conductance of the steel-finned to wooden-finned plates is plotted as a function of distance along plate and weight rate per unit flow area. Within the range of the tests, the maximum value of this ratio was about 1.5 for $y_o = 5/8$ inch and about 2.5 for $y_o = 5\frac{1}{4}$ inches (disregarding data points for the first condensate section). As in the case of added turbulence, the effect of extended surfaces also decreases with the weight rate per unit cross-sectional area. Again this variation can be explained analytically by consideration of equations (7), (14), and (12).

From examination of figures 16 to 19, it may be concluded that about 30 percent (or less) of the over-all effect of finning is due to increase in turbulence and that the remainder is accounted for by the added heat-transfer area. Also when the first point near the leading edge of the plates is disregarded, all these effects of increased heat transfer are almost independent of the distance along the plate.

FLOW PHENOMENA OVER FINNED PLATES

Velocity and temperature distributions.- In figures 12, 13, 20, and 21, are shown the variations of the velocity and temperature distributions with distance at a mean velocity of approximately 30 feet per second for a duct width of $3\frac{1}{8}$ inches. As stated in an earlier part of this discussion, these variations must be known in order to predict the heat transfer from the finned plates. In figure 20, an equation (reference 9), derived for the velocity distribution in the downstream region of a straight circular pipe, is compared with the data taken for the straight rectangular duct. The agreement is well within 10 percent.

Pressure drop.- In figures 7 and 8, the pressure-drop data for the three test plates at duct widths of 5/8, 7/8, $1\frac{5}{8}$, $3\frac{1}{8}$, and $5\frac{1}{4}$ inches are presented. The data were taken under both isothermal and nonisothermal conditions. The change in temperature of the fluid was so small that the flow was almost isothermal and, consequently, there was little difference between the experimental values of the pressure drop for isothermal and nonisothermal flow. Prediction of the pressure drop was made only for isothermal conditions but prediction for nonisothermal flow can be made using equations presented in reference 1. The predicted values of the pressure drop for the finned plate at $y_o = 5/8$ inch differ greatly from the data of references 5, 6, and 7; this difference was probably due to the wall effects of the system.

In all but one instance, the prediction of pressure drop for the case when the unfinned flat plate was in the duct agrees to within 20 percent of the experimental results. For $y_o = 5\frac{1}{4}$ inches, however, the prediction of the pressure drop in the duct is high, coinciding with the data on the pressure drop along the finned plates. The deviation is probably due to the fact that, in the present instrumentation for measuring the pressure drop for $y_o = 5\frac{1}{4}$ inches, the experimental accuracy is of the order of magnitude of the measured values.

The static pressure is plotted against the hydraulic diameter in figure 8 for a constant value of $W = 3500$ pounds of air per hour. When the hydraulic diameter can be taken to be twice the duct width $(2y_o)$, the predicted slope of the curve is -3.0. This value compares well with the measured value of -3.6 in this region.

As stated earlier, the measurements of pressure drop and velocity distribution should be compared with other data taken on "roughened" systems.

A comparison of the added heat transfer and pressure drop upon addition of steel fins on the unfinned plates reveals that the heat transfer is increased from 2 to $3\frac{1}{2}$ times (depending on the value of G) while the pressure drop is increased from 5 to 15 times (depending on the duct width).

CONCLUSIONS

From an experimental study of the thermal and hydrodynamical behavior of air flowing over finned plates, the following conclusions were drawn:

1. The data tend to indicate that the principal criterion for the application of the equations for heat transfer along unfinned flat plates is the initiation of a thermal boundary layer. Thus, in predicting the local value of heat transfer, equation (6) (for flow of air over flat plates) is applicable for $0 < x/D_H < 8.8$ but, for $x/D_H > 8.8$, equation (7) (for flow of air in ducts) may approximate the actual conditions more closely.

2. In the downstream region, the effective thermal conductance for pin-finned plates can be predicted within 35 percent by the use of equations (11) and (12).

3. The over-all effect of finning consists of two parts: (a) The increase of turbulence along the unfinned portion of the plate due to the presence of the fins and the change in velocity distribution between the fins; and (b) the added heat-transfer area of the pin fins. The results of the tests show that for a duct width of 5/8 inch, about 20 percent of the over-all effect is due to the increase of turbulence at G = 20,000 pounds per hour per square foot.

4. The over-all effect of finning and each of its two components decrease with increasing weight rate per unit area as required by the equations used for the prediction of the heat transfer.

5. Both the velocity and temperature distributions vary along the pin-finned test sections. Prediction of the variation of the effective thermal conductance with the distance along the plate in the inlet region (region where the inlet effect is not negligible) and its variation with the duct width requires that further investigation be made of the variation of the velocity distribution with the distance along the plate and the duct width as parameters.

6. Experimental results indicate that much of the air passing along the plate between the fins initially bypasses the fins and flows in the open portion of the duct, where the resistance to flow is less.

7. The addition of steel fins on an unfinned plate caused increased heat-transfer ratios from 2 to $3\frac{1}{2}$ times, while the static-pressure drop was increased 5 to 15 times.

Department of Engineering
University of California
Berkeley, Calif., January 3, 1946

APPENDIX A

SYMBOLS

The following symbols are used in this report:

A	heat-transfer area, sq ft
A_{cs}	cross-sectional area of duct, sq ft
A_F	cross-sectional area of fin in plane parallel to base, sq ft
A_u	unfinned surface area of test section, sq ft
a	transverse pitch in tube bank, diam
b	longitudinal pitch in tube bank, diam
c_p	heat capacity of air at mixed-mean temperature, Btu/(lb)(°F)
D_H	hydraulic diameter $(4A_{cs}/P)$, ft ($D_H = 2y_o$ for small values of y_o)
d	diameter of fins, ft
$\left(\frac{du}{dy}\right)_{y=0}$	velocity gradient at surface of plate, (ft/sec)/ft
F_a	tube-arrangement modulus for flow over tube bank
f_c	unit thermal conductance, Btu/(hr)(sq ft)(°F)
f_E	unit thermal conductance over ends of fins, Btu/(hr)(sq ft)(°F)
f_F	unit thermal conductance along fin, Btu/(hr)(sq ft)(°F)
f_u	unit thermal conductance along unfinned surface, Btu/(hr)(sq ft)(°F)
$(fA)_e$	effective thermal conductance per vertical row of 12 fins and associated plate, Btu/(hr)(°F)(row)

$(fA)_u$	thermal conductance of unfinned area, Btu/(hr)(°F)
G	weight rate per unit of cross-sectional area normal to flow, lb/(hr)(sq ft)
g	gravitational force per unit mass, lb/(lb)(sec²/ft)
Δh_{vap}	heat of vaporization of water at atmospheric pressure, Btu/lb
K	friction factor for flow over tube banks, defined by equation: $\Delta p = \frac{KNG^2}{10.84 \times 10^8 \gamma}$
k	thermal conductivity of fin material, Btu/(hr)(sq ft)(°F/ft)
L	length of fin projecting into air stream, ft
l	over-all heated length from leading edge measured along test section, ft
l_E	equivalent length of end of fin, ft
l_s	length of test section, ft
i	number of vertical rows of fins per condensing section
m_x	fin parameter, defined under equation (12), 1/ft
N	rows of fins in direction of air flow
n	number of fins per vertical row
P	perimeter of fin measured on plane parallel to base of fin; also, heat-transfer perimeter of duct, ft
Δp	static-pressure drop along test section, in. of H_2O
q	heat gained by air stream, Btu/hr
q_r	heat transferred per row from condensing steam in steam chest to air flowing in duct, Btu/(hr)(row)
R	rate of condensation per condensate section under "load" conditions, lb/hr
R'	rate of condensation under "no-load" conditions, lb/hr
Re	Reynolds modulus

Re_d	Reynolds modulus based on diameter of fins
T	mixed-mean air absolute temperature, °R
T_f	arithmetic average of mixed-mean absolute temperature of air and absolute temperature of plate, °R
t_p	plate temperature, °F
u_m	mean fluid velocity, ft/sec
V	air velocity, ft/sec
W	weight rate of air, lb/hr
x	distance along plate from heated leading edge, ft
y	distance from plate, in.
y_o	duct width, in.
γ	weight density of fluid, lb/cu ft
ζ	friction factor, defined by equation: $\frac{5.2(\Delta p)}{\gamma} = \zeta \frac{l_s}{D_H} \frac{u_m^2}{2g}$
$\Delta\tau$	increase in mixed-mean temperature of air flowing past plate, °F
τ_o	temperature of air entering test section, °F
τ_r	mixed-mean temperature of air at each of rows of test plate, °F

Subscripts:

av	average
max	maximum
x	point or local
FP	flat plate
s	steel
w	wooden

APPENDIX B

SAMPLE CALCULATION

The following example is a sample calculation for predicting the heat transfer from the steel-finned plate:

Experimental conditions are:

Type of plate	Steel pin-finned
Air weight rate per unit area, G, lb/(hr)(sq ft).	31,000
Duct width, y_o, in.	$1\frac{5}{8}$
Plate temperature, t_p, ^{o}F	212
Average air temperature at inlet, $\tau_{o_{av}}$, ^{o}F	90

The downstream value of $(fA)_{e_x}$ is to be determined.

From equation (12):

$$(fA)_{e_x} = n\sqrt{f_{F_x}PkA_F}\left[\frac{\sinh m_xL + \frac{f_E}{km_x}\cosh m_xL}{\cosh m_xL + \frac{f_E}{km_x}\sinh m_xL}\right] + A_u f_{u_x}$$

$$f_{u_x} = 5.4 \times 10^{-4}\ T_f^{0.3}\ \frac{G^{0.8}}{D_H^{0.2}}$$

$$f_E = 0.914 \times 10^{-3}\ T_f^{0.3}\ \frac{G^{0.8}}{l_E^{0.2}}$$

$$f_{F_x} = 14.5 \times 10^{-4}\ F_{a_x}T_f^{0.43}\ \frac{G^{0.6}}{d^{0.4}}$$

and

$$k = 26 \text{ Btu/(hr)(sq ft)(}^{\circ}\text{F/ft)}$$

$$d = \frac{5}{16 \times 12} = 0.026 \text{ ft}$$

$$A_F = \pi \frac{d^2}{4} = \pi\left(\frac{5}{16 \times 12}\right)^2 \frac{1}{4} = 0.00053 \text{ sq ft}$$

$$P = \pi d - \pi \frac{5}{16 \times 12} = 0.0817 \text{ ft}$$

$$L = \frac{5}{8 \times 12} = 0.052 \text{ ft}$$

$$l_E = \frac{d}{\sqrt{2}} = \frac{5}{16 \times 12 \times 1.416} = 0.0183 \text{ ft}$$

$$A_u = \left(\frac{1}{12} \times 1\right) - (12 \times 0.00053) = 0.0769 \text{ sq ft}$$

$$D_H = \frac{4A}{P} = \frac{4 \times \frac{13}{8} \times 12}{\left(24 + \frac{26}{8}\right)12} = 0.238 \text{ ft}$$

$$T_f = \frac{212 + 90}{2} = 151^{\circ} \text{ F} = 611^{\circ} \text{ R}$$

$$F_{a_x} = 1.43$$

Thus

$$f_{u_X} = 5.4 \times 10^{-4}(611)^{0.3} \frac{(31,000)^{0.8}}{(0.238)^{0.2}} = 19.3 \text{ Btu/(hr)(sq ft)(}^{\circ}\text{F)}$$

$$f_E = 0.914 \times 10^{-3}(611)^{0.3} \frac{(31,000)^{0.8}}{(0.0183)^{0.2}} = 54.3 \text{ Btu/(hr)(sq ft)(}^{\circ}\text{F)}$$

$$f_{F_X} = 14.5 \times 10^{-4} \times 1.43(611)^{0.43} \frac{(31,000)^{0.6}}{(0.026)^{0.4}} = 70.0 \text{ Btu/(hr)(sq ft)(}^{\circ}\text{F)}$$

$$m_X = \sqrt{\frac{f_{F_X}P}{kA_F}} = \sqrt{\frac{70 \times 0.0817}{26 \times 0.00053}} = 20.4$$

$$\sqrt{f_{F_X}PKA_F} = 0.281$$

Then

$$(fA)_{eX} = 12 \times 0.281\left(\frac{\sinh 20.4 \times 0.052 + \frac{54.3}{26 \times 20.4}\cosh 20.4 \times 0.052}{\cosh 20.4 \times 0.052 + \frac{54.3}{26 \times 20.4}\sinh 20.4 \times 0.052}\right) +$$

$$19.3 \times 0.0769$$

$$= 2.77 + 1.48$$

$$= 4.25 \text{ Btu/(hr)(}^{\circ}\text{F)(row)}$$

From table IV the measured value of $(fA)_{eX}$ is 3.5 Btu/(hr)($^{\circ}$F)(row).

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TABLE I

LOCAL AND AVERAGE EFFECTIVE THERMAL CONDUCTANCES OVER STEEL-FIN PLATE AS A FUNCTION OF WEIGHT RATE PER UNIT AREA AND DUCT WIDTH

Run	W (lb/hr)	G (lb/(hr)(sq ft))	y_o (in.)	$(fA)_e$ (Btu/(hr)(°F)(row)) Local, x (in.) 1.3	2.3	3.3	4.3	5.8	7.8	10.3	13.8	Average
37	945	2,240	$5\frac{1}{4}$	1.60	0.96	0.92	0.89	0.70	0.67	0.58	0.60	0.75
30	2180	5,180	$5\frac{1}{4}$	2.44	1.54	1.54	1.38	1.17	1.07	.96	1.01	1.22
31	3510	8,340	$5\frac{1}{4}$	3.11	2.04	2.01	1.84	1.50	1.45	1.25	1.33	1.60
32	5160	12,200	$5\frac{1}{4}$	3.95	2.71	2.59	2.42	2.02	1.85	1.67	1.79	2.04
27	925	3,790	$3\frac{1}{8}$	1.97	1.20	1.23	1.17	.97	.96	.85	.91	1.04
28	2200	9,020	$3\frac{1}{8}$	3.28	2.13	2.02	2.00	1.60	1.53	1.40	1.38	1.69
38	3500	14,300	$3\frac{1}{8}$	4.31	2.92	2.76	2.72	2.30	2.12	1.91	1.95	2.34
29	5250	21,500	$3\frac{1}{8}$	5.01	4.02	3.24	3.08	2.78	2.76	2.45	2.54	2.93
28	938	7,870	$1\frac{5}{8}$	2.93	1.86	1.91	1.91	1.69	1.58	1.44	1.48	1.69
25	2130	17,850	$1\frac{5}{8}$	4.42	2.94	3.06	2.39	2.51	2.69	2.34	2.34	2.64
26	3480	29,200	$1\frac{5}{8}$	5.65	3.63	3.80	3.62	3.34	3.68	3.20	3.31	3.57
27	5140	43,100	$1\frac{5}{8}$	6.28	4.24	4.40	3.76	4.12	4.55	4.14	3.90	4.27
36	530	14,800	$\frac{5}{8}$	3.35	2.44	2.40	2.54	2.30	2.31	2.24	2.37	2.41
35	940	26,200	$\frac{5}{8}$	4.66	3.50	3.43	3.59	3.08	3.32	3.20	3.21	3.36
35a	950	26,600	$\frac{5}{8}$	4.78	3.74	3.54	3.71	3.42	3.43	3.30	3.42	3.54
34	2140	59,800	$\frac{5}{8}$	7.23	5.48	5.40	5.60	5.22	5.19	5.02	5.20	5.36
33	3420	95,500	$\frac{5}{8}$	9.60	7.36	7.18	7.37	6.58	6.86	6.52	6.83	7.02

TABLE II

LOCAL AND AVERAGE EFFECTIVE THERMAL CONDUCTANCES OVER WOODEN-FIN PLATE AS A FUNCTION OF WEIGHT RATE PER UNIT AREA AND DUCT WIDTH

Run	W (lb/hr)	G (lb/(hr)(sq ft))	y_o (in.)	$(fA)_e$ (Btu/(hr)(°F)(row)) Local x (in.) 1.3	2.3	3.3	4.3	5.8	7.8	10.3	13.8	Average
58	980	27,000	$\frac{5}{8}$	3.03	2.38	2.38	2.37	2.27	2.19	2.13	2.17	2.27
56	2170	60,200	$\frac{5}{8}$	5.00	4.09	4.13	4.09	3.94	3.59	3.55	3.64	3.83
57	3470	96,300	$\frac{5}{8}$	7.04	5.87	5.71	5.66	5.23	5.00	4.78	4.88	5.24
52	942	16,600	$\frac{7}{8}$	1.61	1.20	1.13	1.05	.93	.85	.99	.96	1.02
53	2100	37,000	$\frac{7}{8}$	2.18	1.87	1.69	1.59	1.31	1.25	1.37	1.46	1.44
54	3480	61,300	$\frac{7}{8}$	3.24	2.56	2.37	2.23	2.08	2.00	2.00	2.20	2.22
55	5000	88,000	$\frac{7}{8}$	4.96	3.28	3.08	2.80	2.72	2.62	2.56	2.89	2.92
71	925	3,810	$3\frac{1}{8}$	----	.72	.57	.52	.44	.41	.52	.50	.51
44	950	3,910	$3\frac{1}{8}$	.38	.48	.42	.44	.29	.27	.46	.41	.39
68	2050	8,440	$3\frac{1}{8}$	.99	1.28	1.07	1.00	.84	.79	.89	.87	.92
46	2090	8,610	$3\frac{1}{8}$	1.08	.93	.90	.81	.65	.56	.78	.86	.79
69	3350	13,790	$3\frac{1}{8}$	1.05	1.83	1.51	1.32	1.21	1.17	1.24	1.34	1.30
45	3500	14,400	$3\frac{1}{8}$	1.66	1.35	1.31	1.12	.81	.85	1.10	1.09	1.09
70	4950	20,350	$3\frac{1}{8}$	1.27	2.35	1.96	1.92	1.59	1.53	1.65	1.63	1.68
47	5060	20,800	$3\frac{1}{8}$	2.55	1.77	1.73	1.46	1.26	1.18	1.39	1.49	1.50
43	930	2,210	$5\frac{1}{4}$	.26	.42	.38	.35	.29	.23	.40	.34	.33
64	965	2,290	$5\frac{1}{4}$	.33	.41	.33	.31	.28	.26	.37	.32	.30
42	1152	2,740	$5\frac{1}{4}$	.28	.47	.40	.38	.24	.19	.42	.38	.34
39	2000	4,750	$5\frac{1}{4}$	.87	.81	.75	.60	.55	.45	.66	.71	.66
67	2050	4,880	$5\frac{1}{4}$	.51	.80	.64	.67	.54	.40	.62	.58	.58
66	3390	8,050	$5\frac{1}{4}$	.74	1.18	.95	.85	.78	.70	.85	.84	.84
40	3480	8,270	$5\frac{1}{4}$	1.41	1.14	1.08	.79	.80	.70	1.03	.96	.95
65	4840	11,500	$5\frac{1}{4}$	1.03	1.50	1.22	1.12	1.02	.95	1.13	1.14	1.12
41	5070	12,000	$5\frac{1}{4}$	1.86	1.43	1.37	1.15	.97	.89	1.16	1.17	1.18

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TABLE III

LOCAL AND AVERAGE EFFECTIVE THERMAL CONDUCTANCES FROM UNFINNED PLATE AS A FUNCTION OF WEIGHT RATE PER UNIT AREA AND DUCT WIDTH

Run	W (lb/hr)	G (lb/(hr)(sq ft))	y_o (in.)	$(fA)_e$ (Btu/(hr)(°F)(row)) Local x (in.) 1.3	2.3	3.3	4.3	5.8	7.8	10.3	13.8	Average
75	890	2,030	$5\frac{1}{4}$	0.57	0.24	0.20	0.20	0	0.15	0.17	0.12	0.167
74	2120	4,840	$5\frac{1}{4}$	.96	.37	.38	.37	.36	.29	.35	.42	.407
73	3430	7,800	$5\frac{1}{4}$	1.46	.65	.59	.56	.52	.46	.56	.67	.639
72	5000	11,400	$5\frac{1}{4}$	1.95	.87	.91	.96	.72	.64	.77	.91	.891
76	900	3,460	$3\frac{1}{8}$	.77	.35	.30	.30	.30	.22	.26	.27	.308
77	2170	8,350	$3\frac{1}{8}$	1.37	.60	.71	.58	.59	.49	.59	.66	.655
78	3400	13,100	$3\frac{1}{8}$	1.84	.88	.93	.76	.76	.67	.85	.97	.913
79	5000	19,200	$3\frac{1}{8}$	2.66	1.25	1.29	1.13	1.09	.98	1.17	1.35	1.292
84	877	6,500	$1\frac{5}{8}$	1.27	.54	.63	.51	.50	.43	.49	.53	.560
81	2070	15,330	$1\frac{5}{8}$	2.30	1.03	1.13	1.97	.95	.89	.91	1.07	1.14
80	3400	25,200	$1\frac{5}{8}$	3.32	1.52	1.67	1.45	1.41	1.30	1.40	1.56	1.59
82	4820	35,700	$1\frac{5}{8}$	4.34	1.97	2.21	1.86	1.74	1.78	1.80	2.06	2.07
83	5000	36,900	$1\frac{5}{8}$	4.41	2.10	2.33	2.01	1.96	1.78	1.98	2.18	2.20
85	870	11,950	$\frac{7}{8}$	2.08	.91	1.02	1.13	.86	.74	.87	.92	.975
86	2078	28,400	$\frac{7}{8}$	3.94	1.91	2.06	1.75	1.74	1.62	1.74	1.93	1.95
87	3330	45,700	$\frac{7}{8}$	4.62	2.28	2.48	2.12	2.30	2.12	2.02	2.45	2.41
88	5050	69,200	$\frac{7}{8}$	4.69	2.93	3.32	2.60	2.74	2.52	2.56	2.67	2.83
c	900	17,300	$\frac{5}{8}$	2.63	1.02	1.11	1.05	.93	.98	.95	1.03	1.11
C	900	17,300	$\frac{5}{8}$	2.10	.98	1.01	1.01	.89	.85	.88	.98	1.01
89	2050	39,400	$\frac{5}{8}$	5.15	2.66	2.78	2.43	2.46	2.30	2.41	2.58	2.67
a	2100	40,300	$\frac{5}{8}$	5.49	2.42	2.50	2.47	2.30	2.29	2.19	2.35	2.54
A	2100	40,300	$\frac{5}{8}$	4.53	2.10	2.21	2.27	1.96	2.03	2.03	2.14	2.25
b	3300	63,400	$\frac{5}{8}$	7.58	3.65	3.69	3.68	3.38	3.41	3.26	3.50	3.73
90	3480	66,800	$\frac{5}{8}$	6.65	3.50	3.73	3.25	3.35	3.22	3.15	3.09	3.47
B	3500	67,000	$\frac{5}{8}$	6.25	3.22	3.29	3.29	3.36	3.00	2.96	3.20	3.63

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TABLE IV

MEASURED AND PREDICTED VALUES OF EFFECTIVE THERMAL CONDUCTANCE IN

DOWNSTREAM REGION[a] OF TEST PLATES[b]

G (lb/(hr)(sq ft))	System	$(fA)_e$ (Btu/(hr)(°F)(row))						
		y_o (in.)						
		5/8		7/8	$1\frac{5}{8}$	$3\frac{1}{8}$	$5\frac{1}{4}$	All
		Measured	Predicted[c]	Measured				Predicted[d]
2,000	Steel-finned plate	[e]0.7	0.77	[e]0.7	[e]0.7	0.6	0.6	0.82
	Wooden-finned plate	[e].4	.28	[e].2	[e].22	.3	.3	.30
	Unfinned plate	[e].16	.17	[e].18	[e].20	.17	.16	.18
	Steel fins only		.62					.65
	Wooden fins only		.13					.13
	$(fA)_u$		.15					.17
5,000	Steel-finned plate	[e]1.2	1.34	[e]1.2	1.2	1.1	1.1	1.44
	Wooden-finned plate	[e].7	.48	[e].4	[e].46	.6	.6	.51
	Unfinned plate	[e].35	.34	[e].38	.40	.36	.35	.37
	Steel fins only		1.03					1.10
	Wooden fins only		.17					.17
	$(fA)_u$		.31					.34
12,500	Steel-finned plate	2.1	2.31	[e]2.1	2.1	1.9	1.9	2.46
	Wooden-finned plate	1.3	.88	.7	[e].82	1.1	1.1	.92
	Unfinned plate	.78	.71	.78	.82	.76	.77	.76
	Steel fins only		1.65					1.76
	Wooden fins only		.22					.22
	$(fA)_u$		.66					.70
31,000	Steel-finned plate	3.5	3.93	[e]3.5	3.5	3.2	[e]3.2	4.25
	Wooden-finned plate	2.4	1.66	1.3	[e]1.5	2.0	[e]2.0	1.77
	Unfinned plate	1.7	1.48	1.64	1.6	1.6	[e]1.67	1.58
	Steel fins only		2.56					2.77
	Wooden fins only		.29					.29
	$(fA)_u$		1.37					1.48

[a]At $x = 12$ in.

[b]The method of calculation is given in the section Prediction of Heat Transfer and Static-Pressure Drop for Rectangular Pin-Finned Duct (Heat transfer) under METHOD OF ANALYSIS. (See also appendix B.)

[c]5/8-in. data calculated with no heat loss from ends of fins postulated.

[d]With average weight rate used.

[e]Extrapolated data.

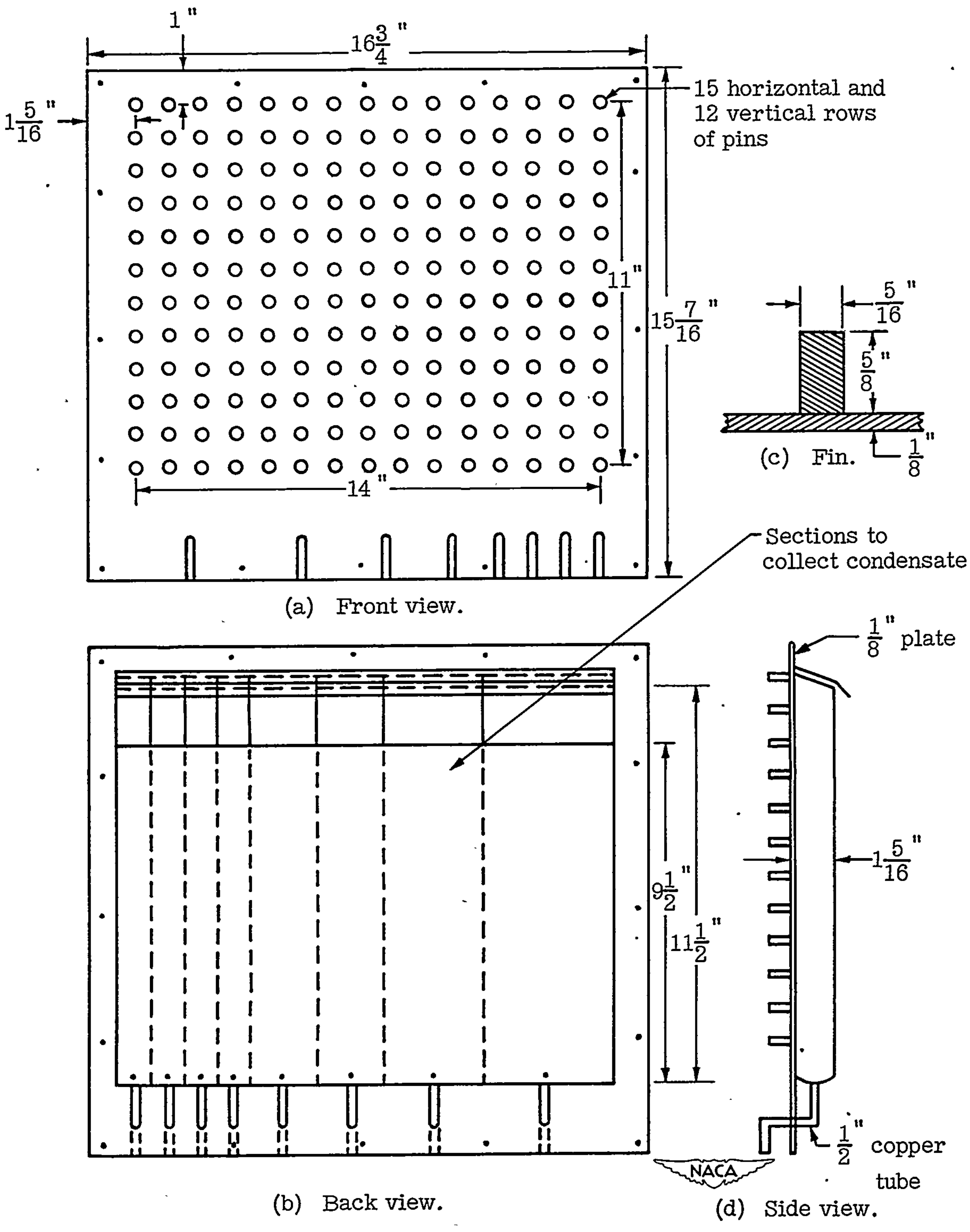


Figure 1.- Pin-fin plate.

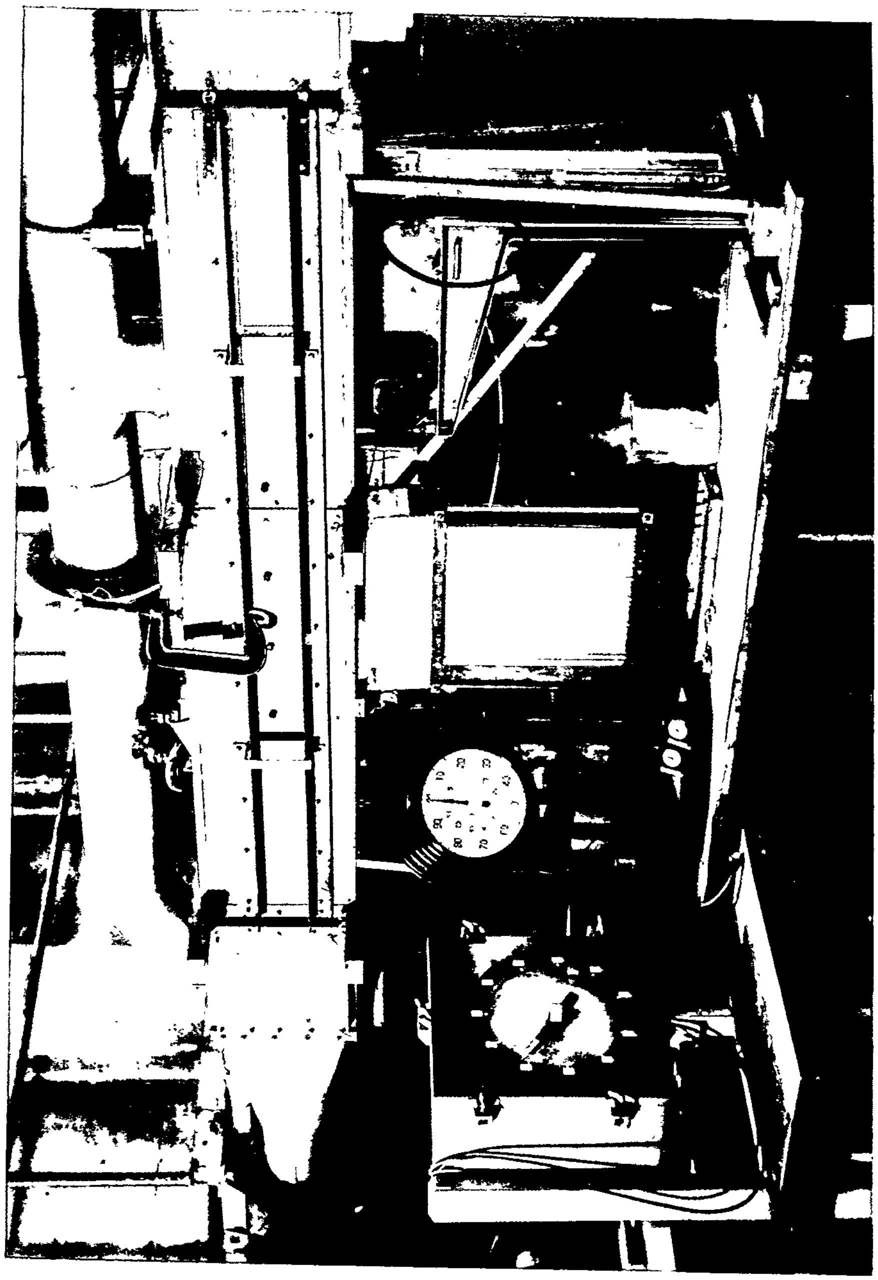


Figure 2.- Pin-fin test stand for the determination of local values of unit thermal conductance.

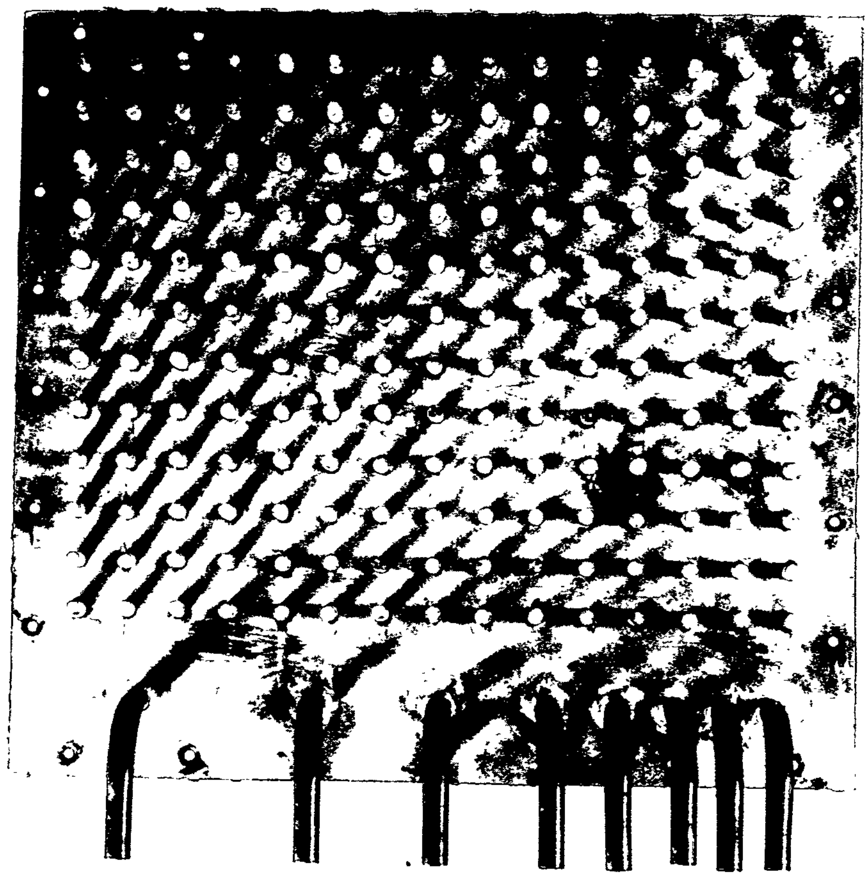


Figure 3.- Pin-fin test elements with wooden pins. An identical section (not shown) was built with steel pin fins and also one with no pins (unfinned flat plate).

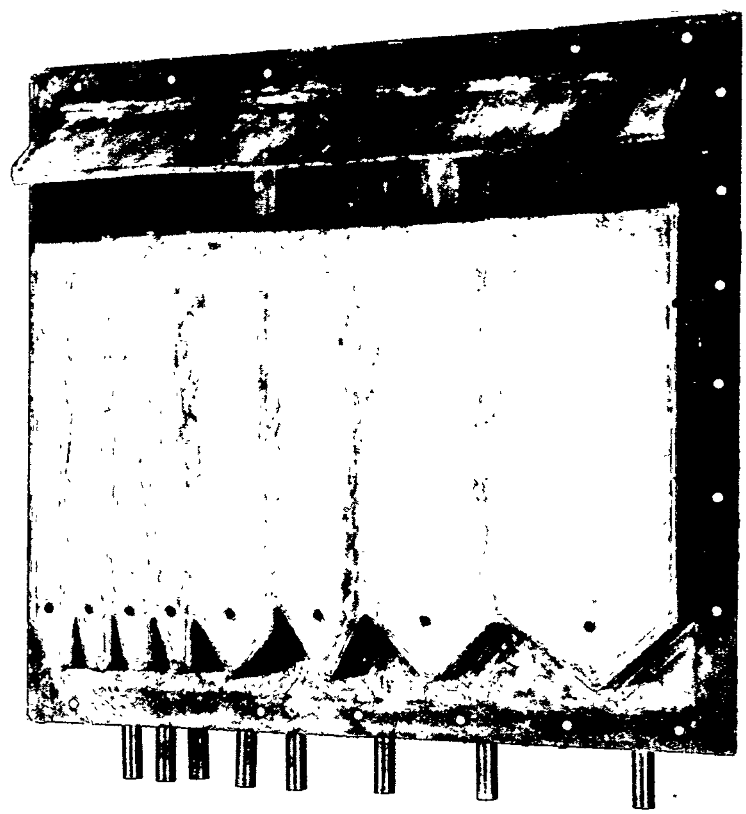


Figure 4.- Typical condensate-collection section on steam side of test plates.

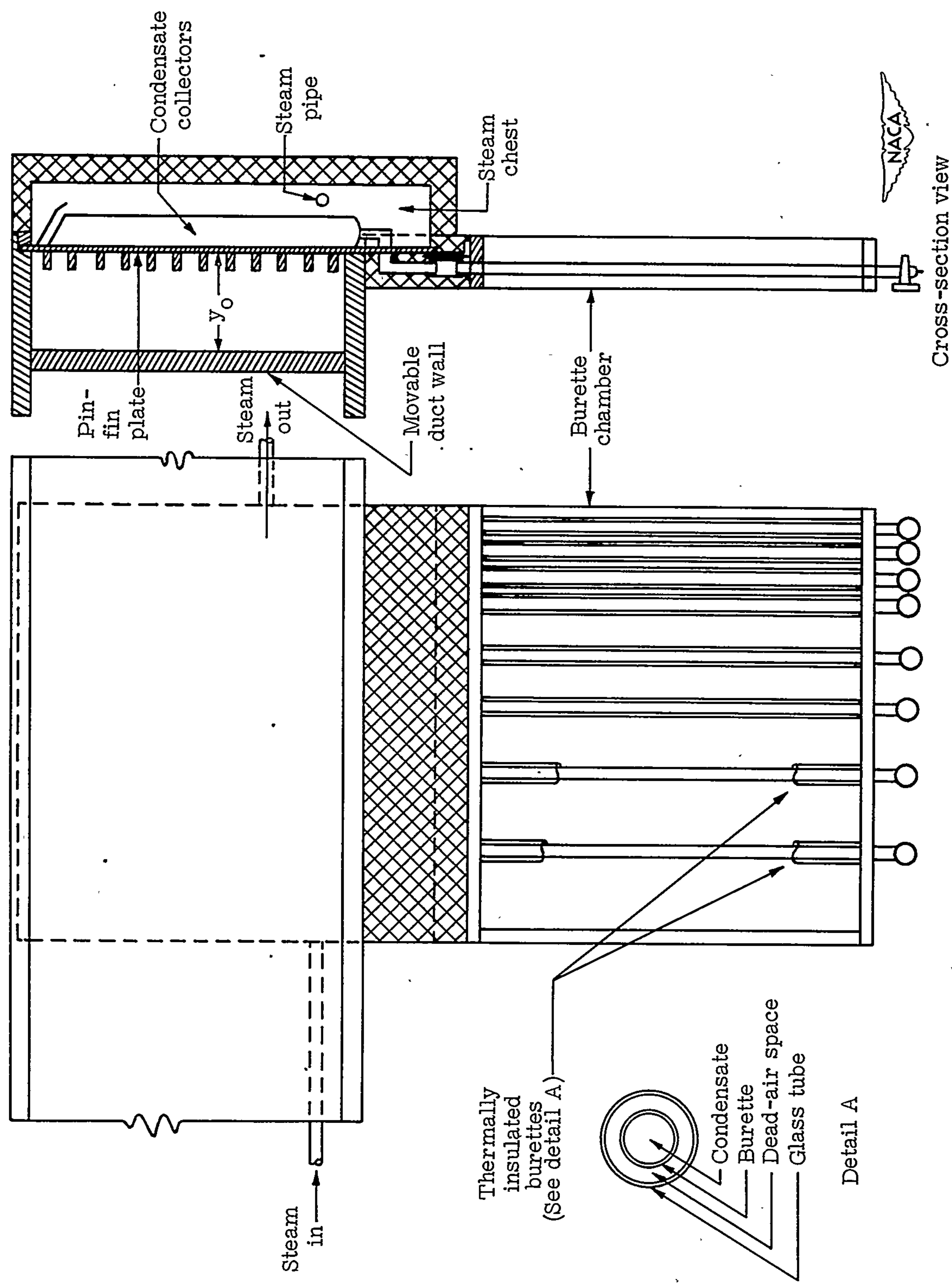


Figure 5.- Pin-fin test section.

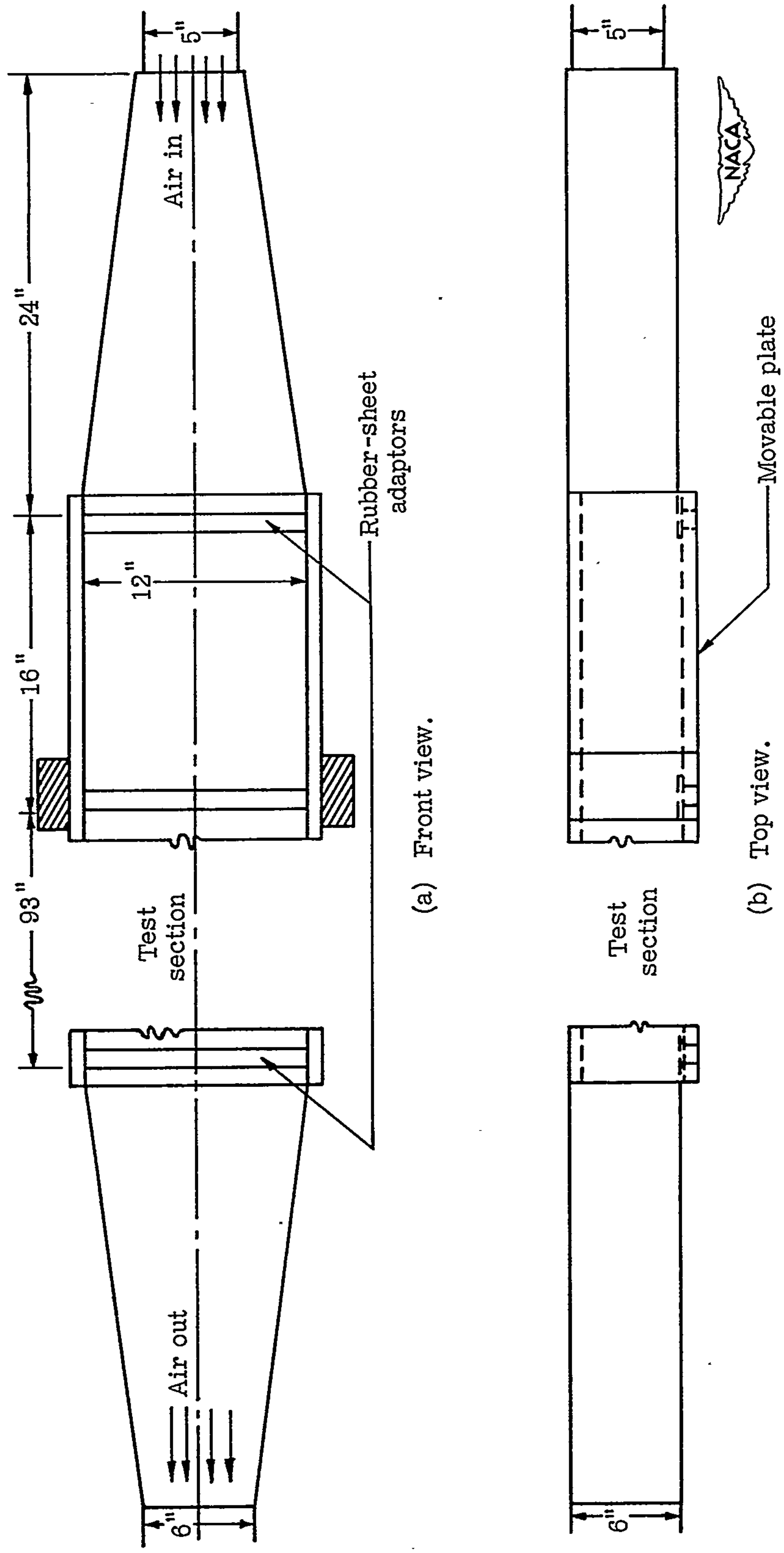
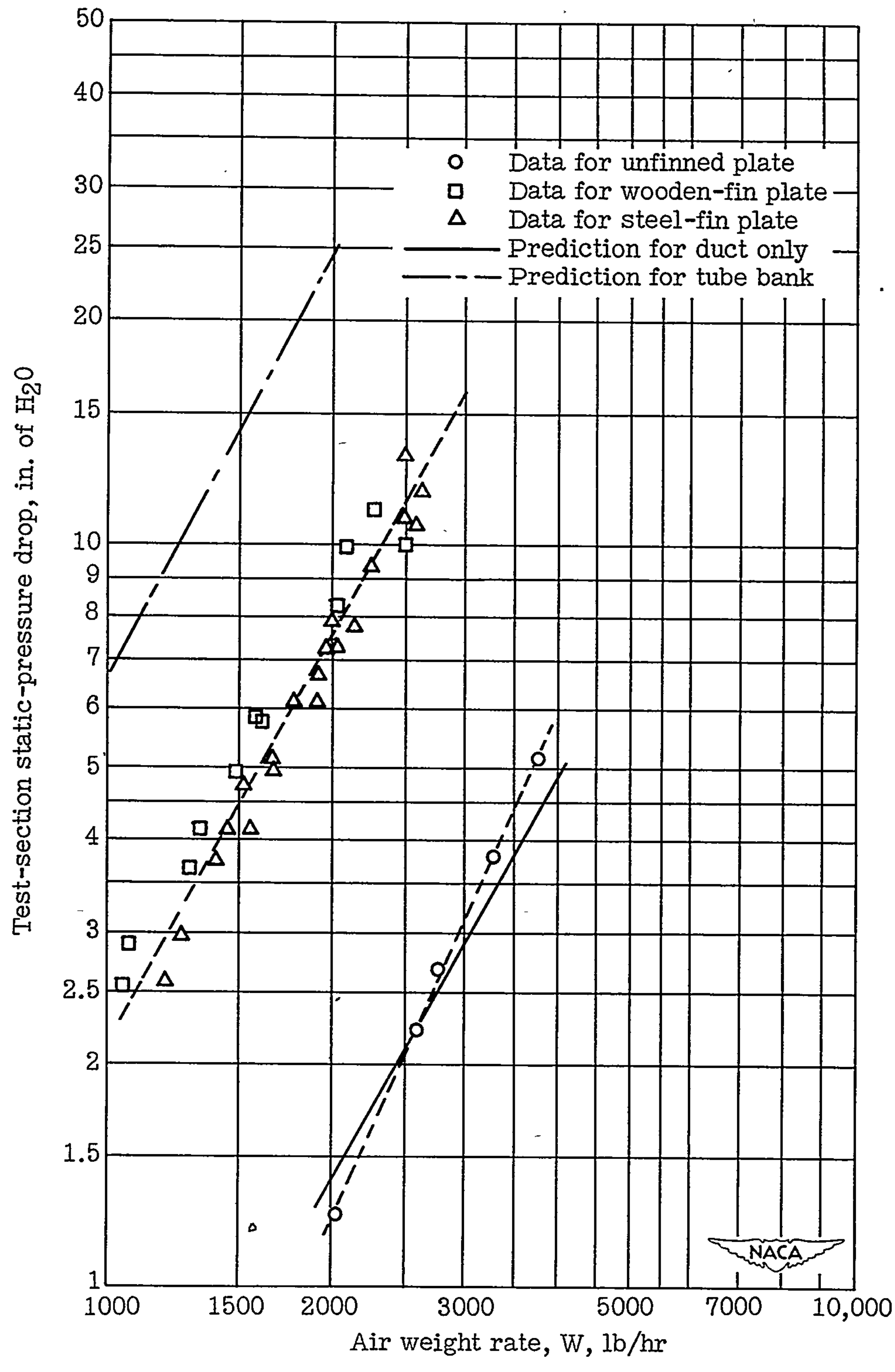
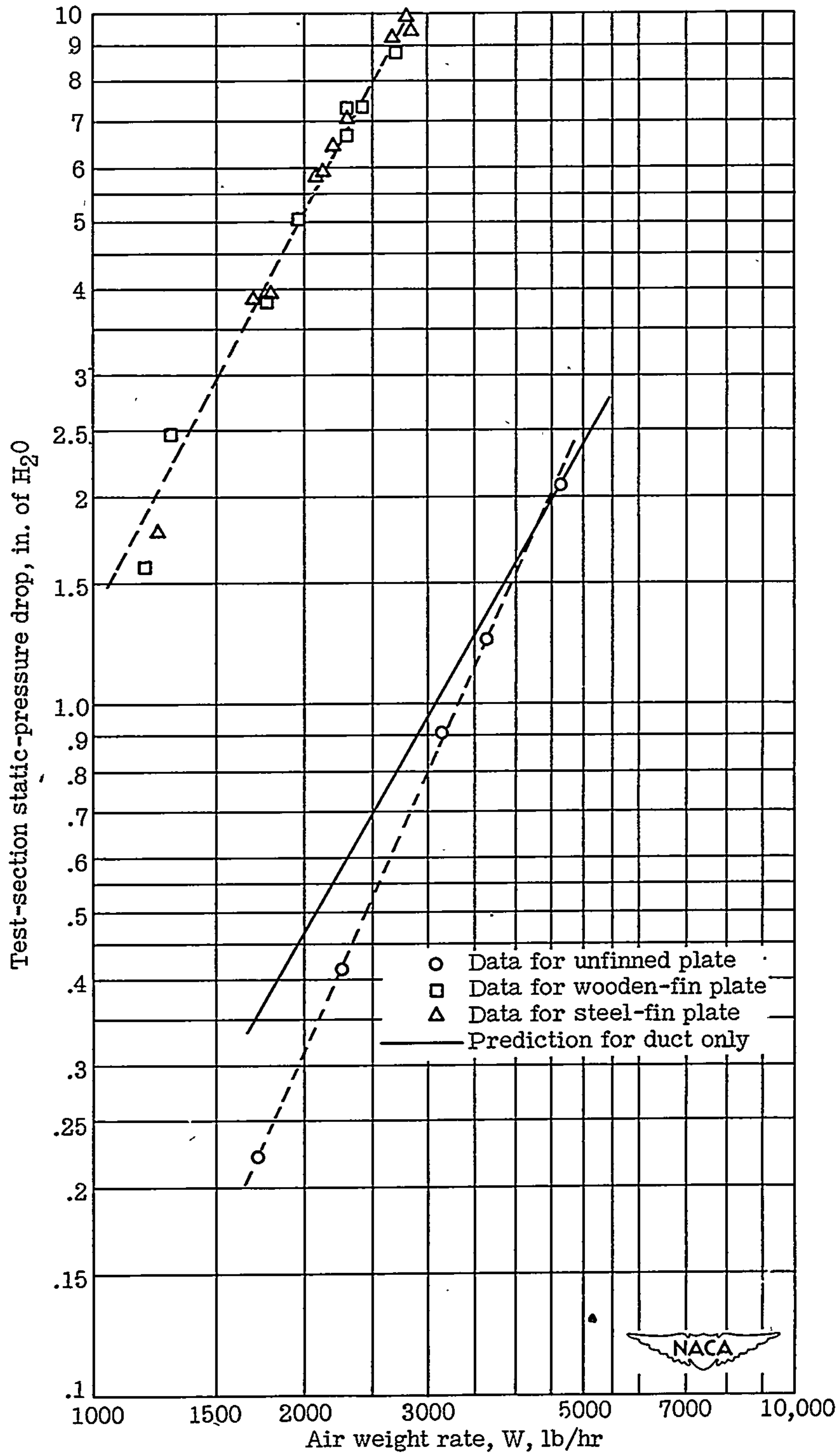


Figure 6.- Variable-width duct housing test section.



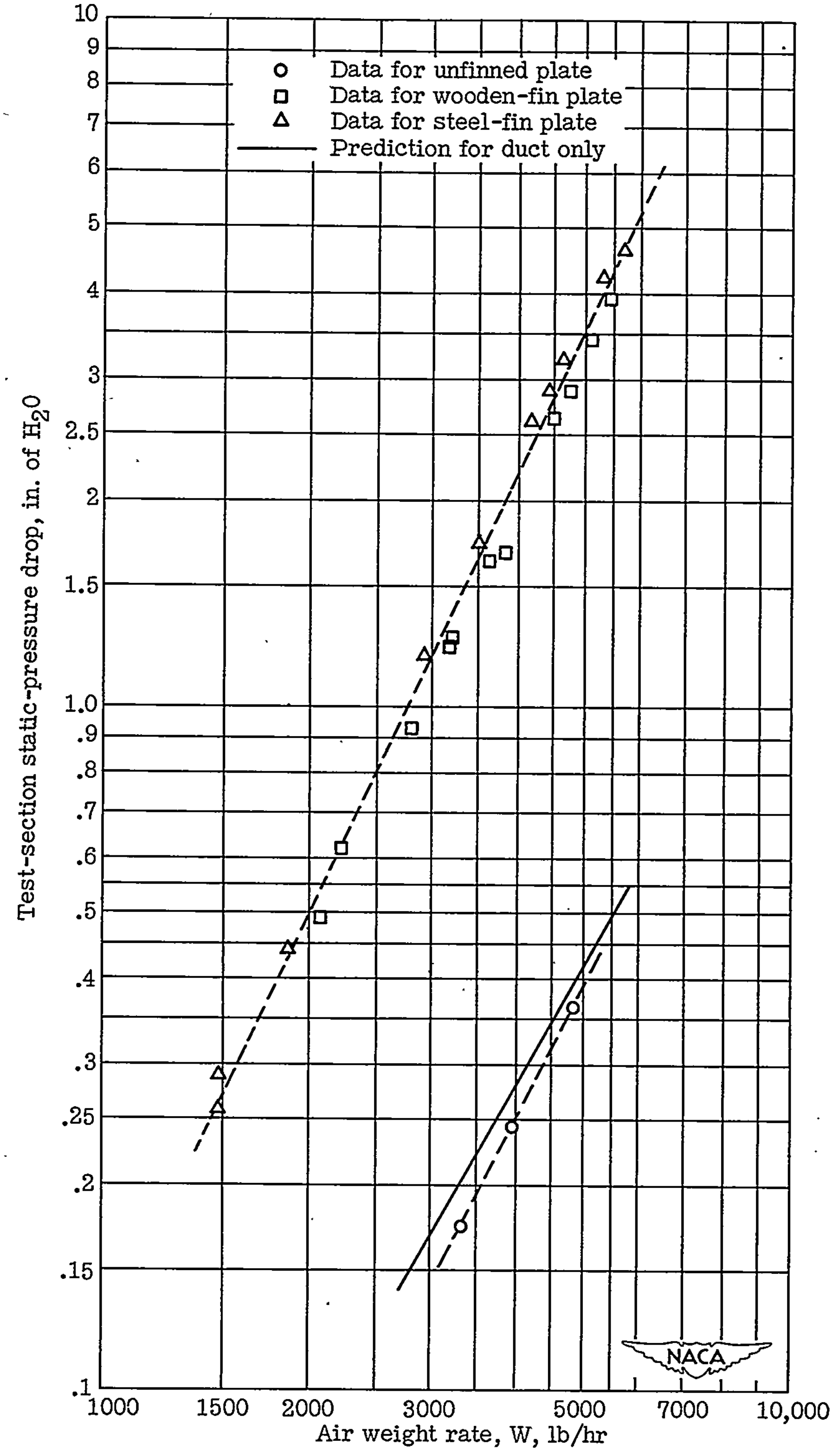
(a) y_o = 5/8 inch.

Figure 7.- Effect on test-section isothermal static-pressure drop of air weight rate.



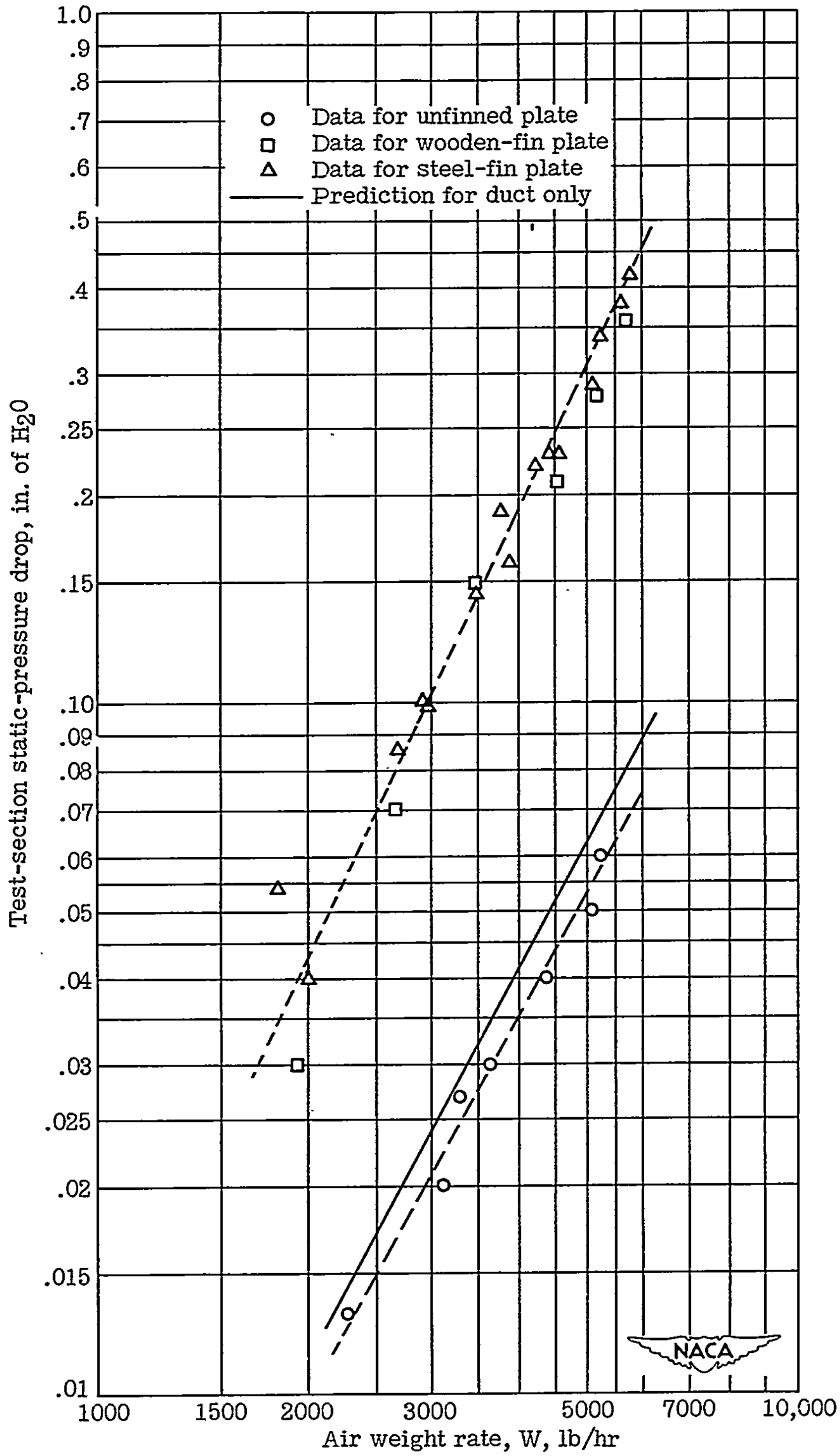
(b) y_o = 7/8 inch.

Figure 7.- Continued.



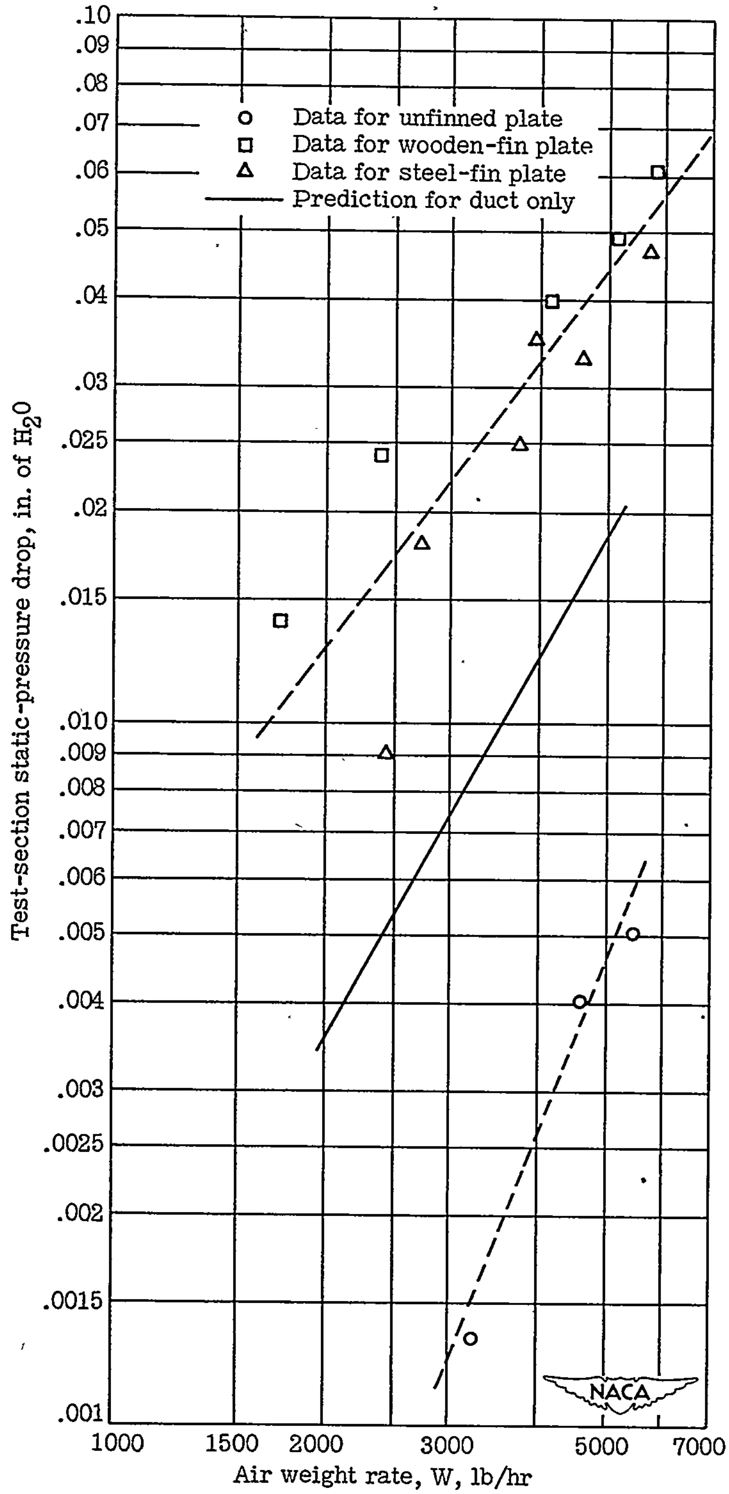
(c) $y_o = 1\frac{5}{8}$ inches.

Figure 7.- Continued.



(d) $y_0 = 3\frac{1}{8}$ inches.

Figure 7.- Continued.



(e) $y_o = 5\frac{1}{4}$ inches.

Figure 7.- Concluded.

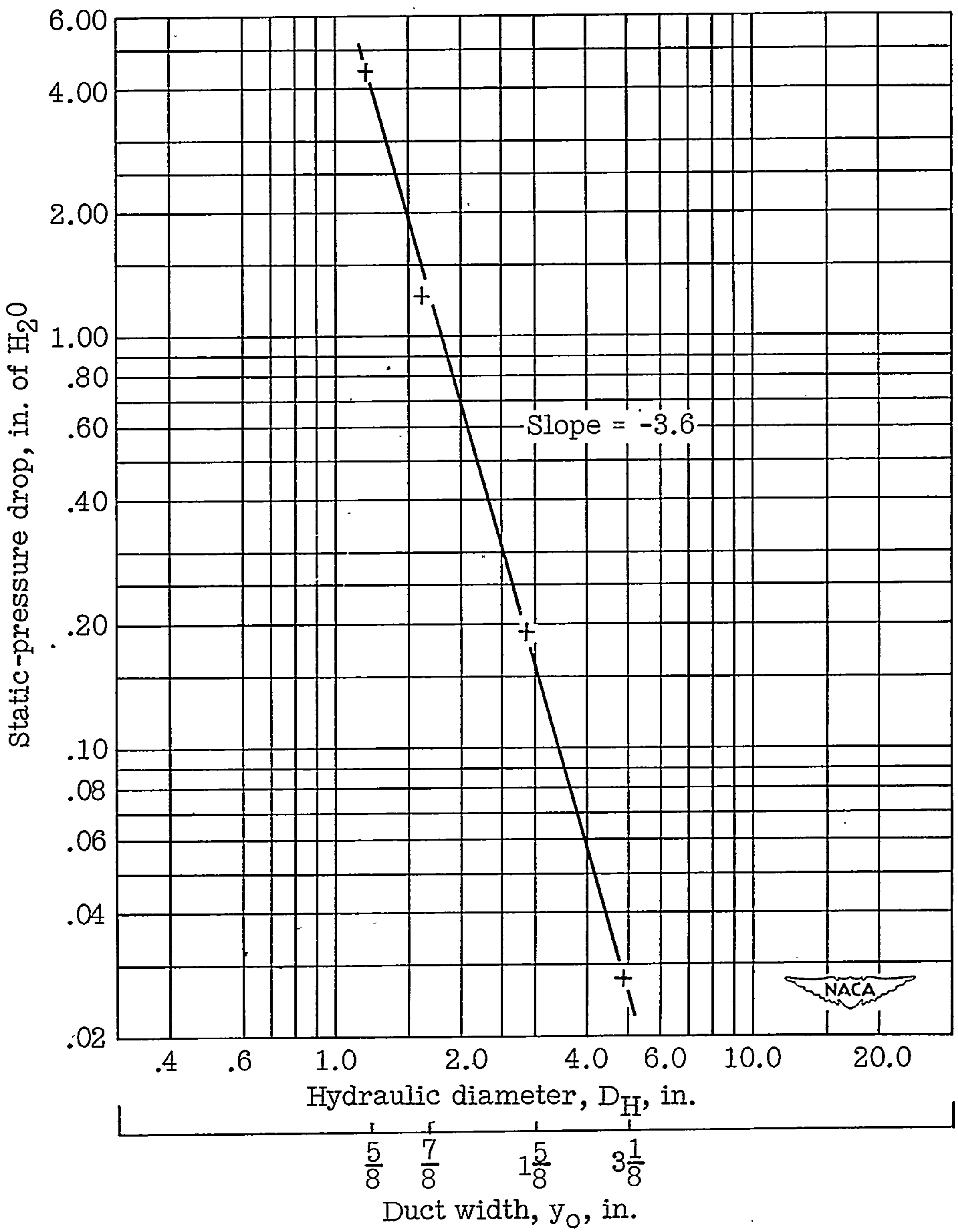
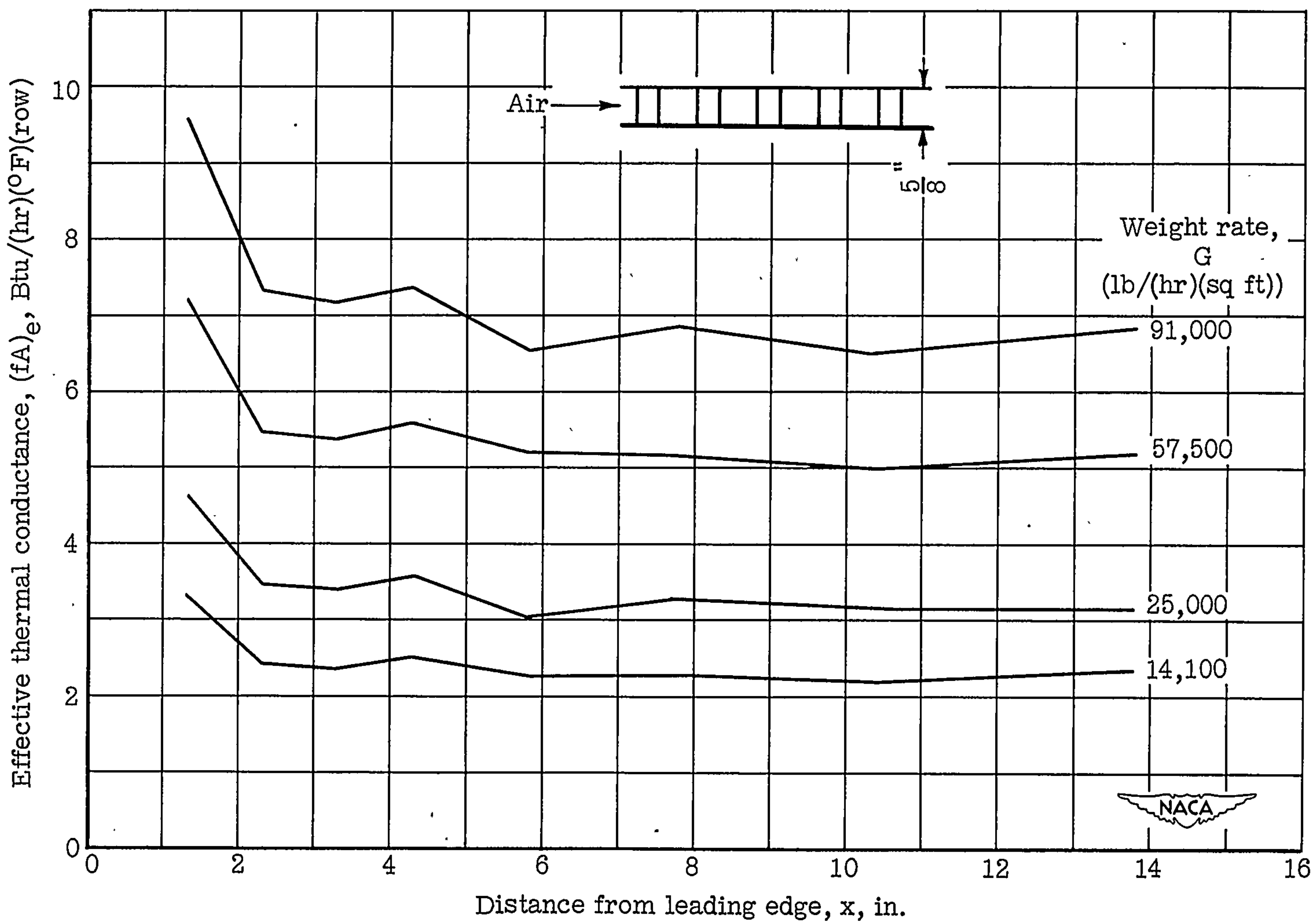
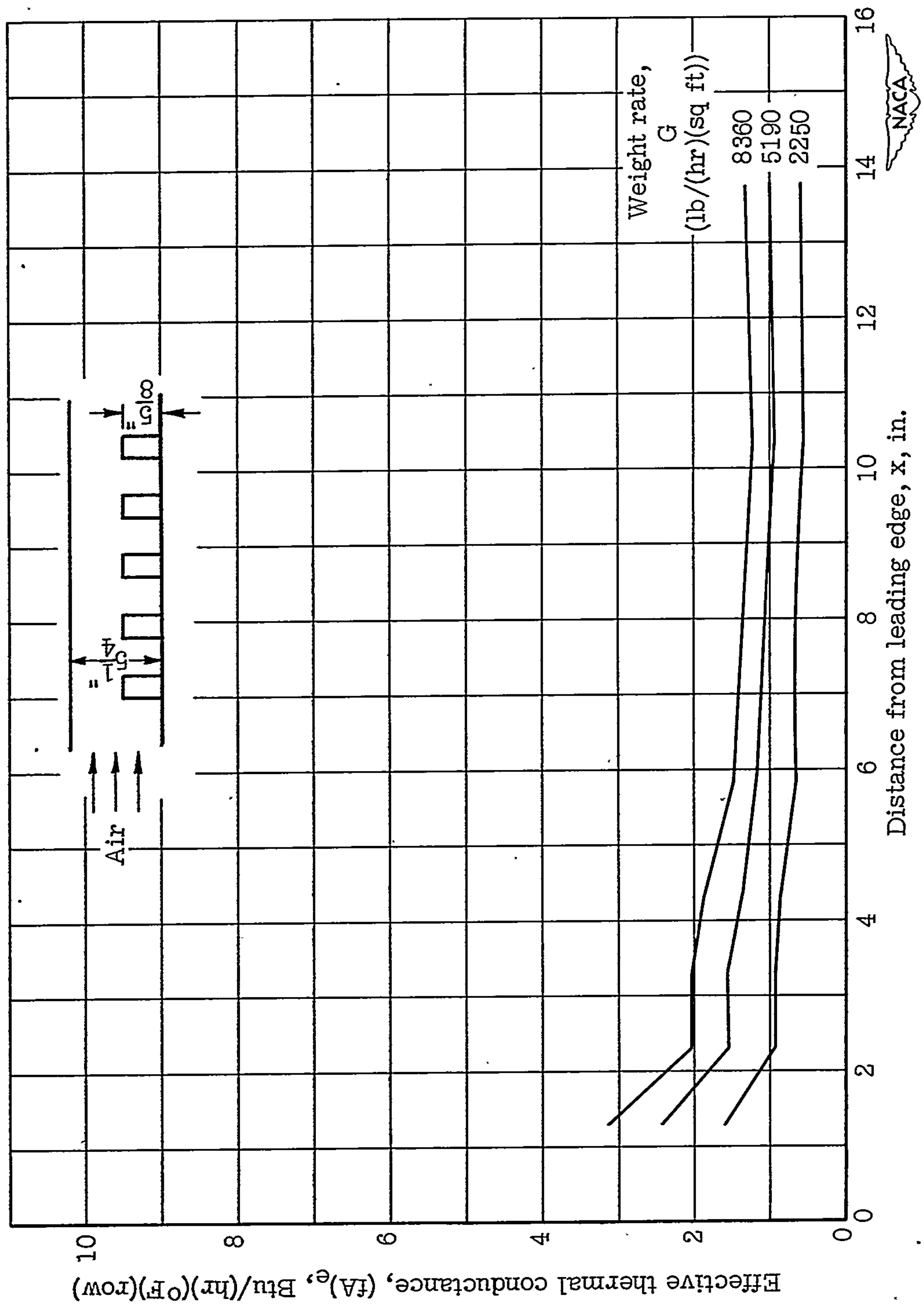


Figure 8.- Effect on static-pressure drop of hydraulic diameter for unfinned flat plate. Air weight rate W, 3500 pounds per hour.



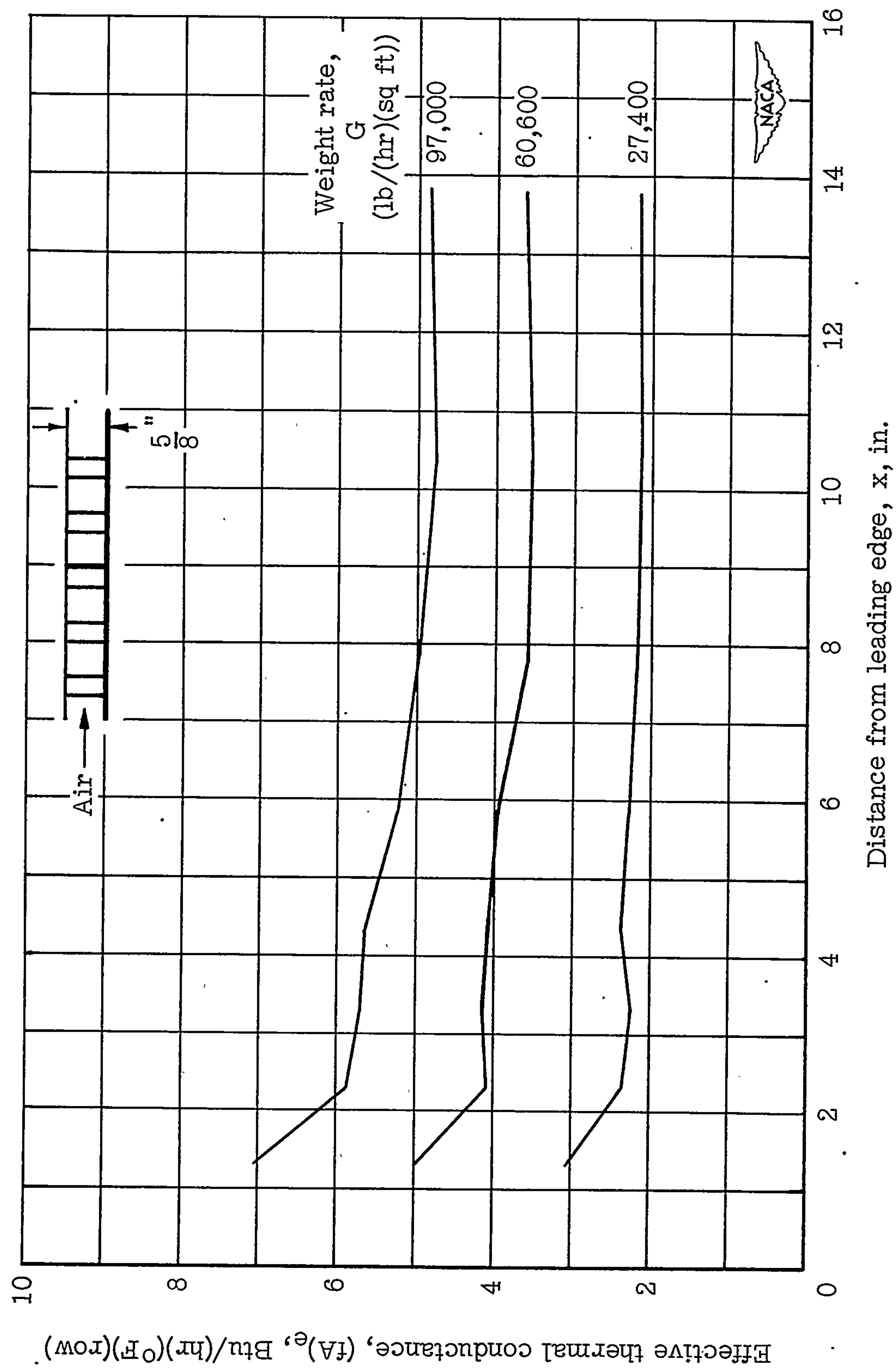
(a) y_o = 5/8 inch.

Figure 9.- Local values of effective thermal conductance from steel-pin-fin test stand.



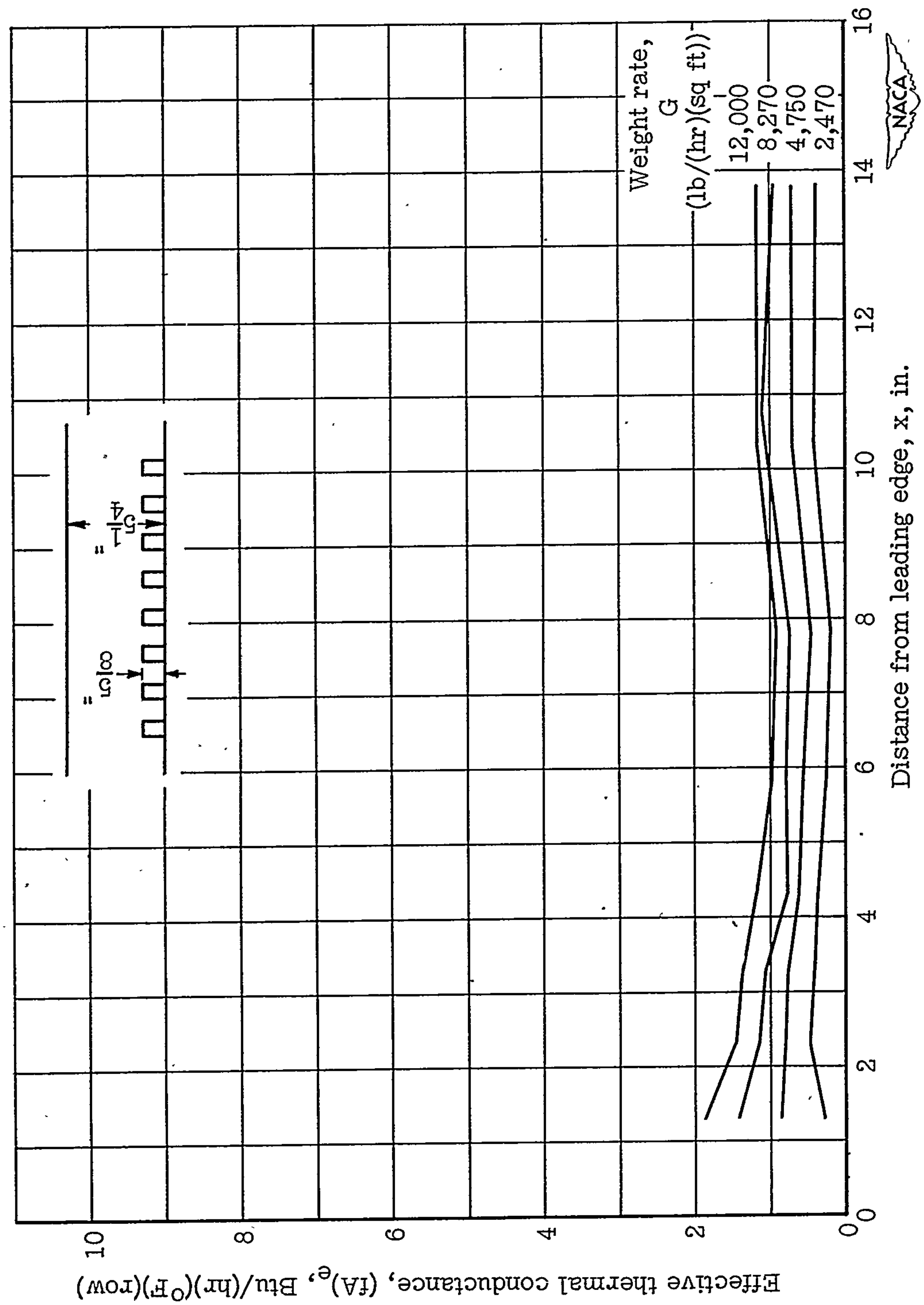
(b) $y_o = 5\frac{1}{4}$ inches.

Figure 9.- Concluded.



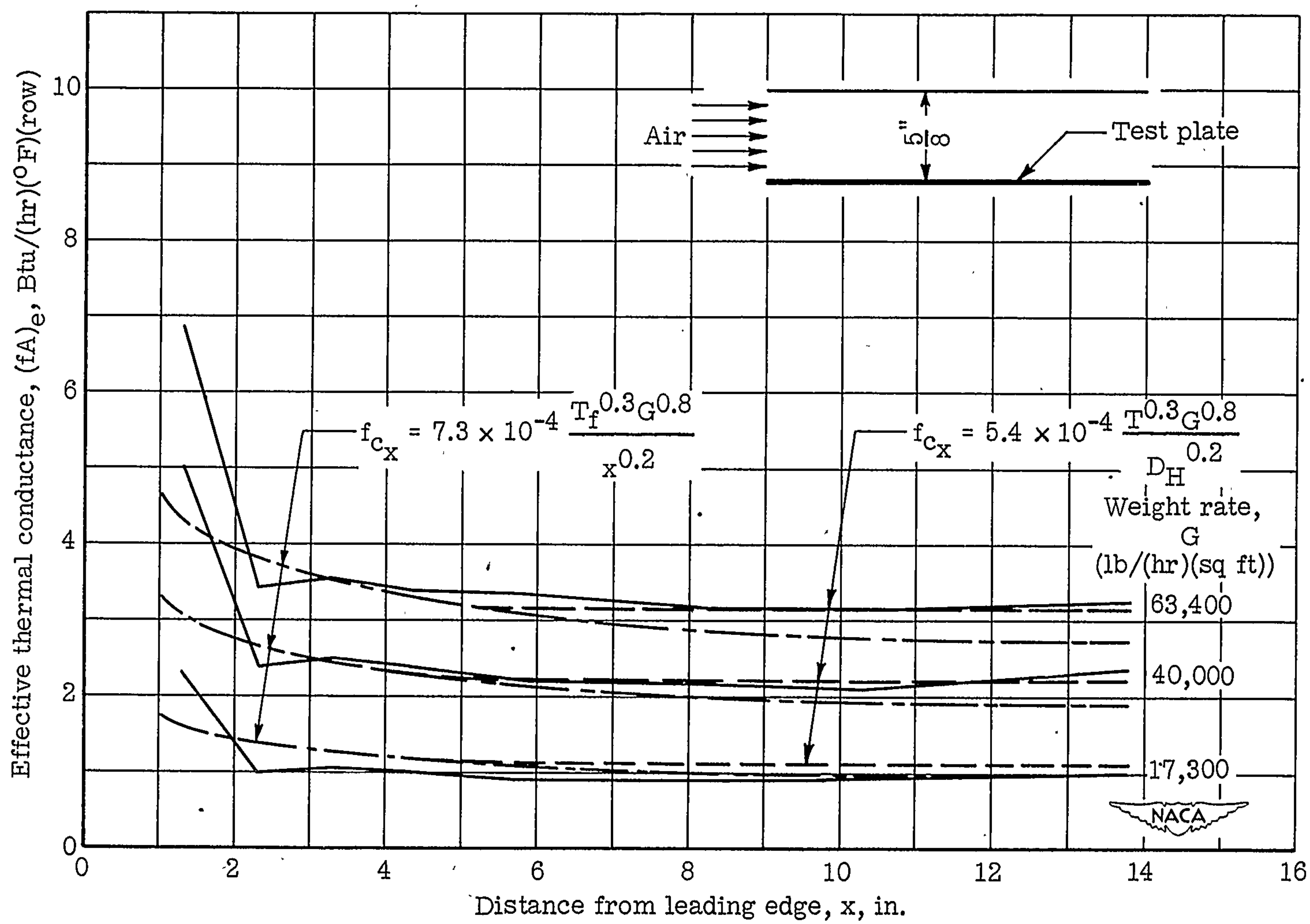
(a) $y_o = 5/8$ inch.

Figure 10.- Local values of effective thermal conductance from wooden-pin-fin test stand.



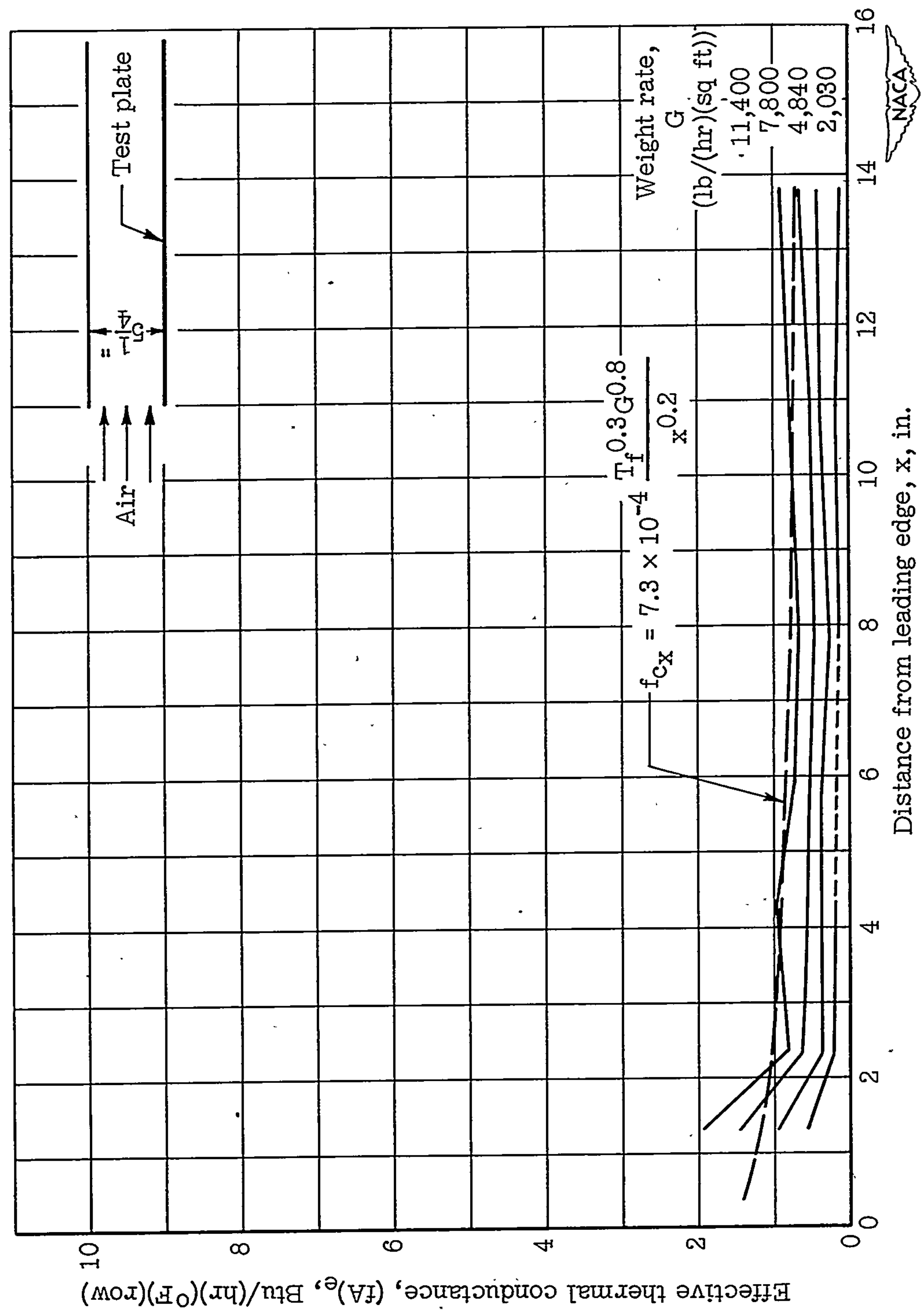
(b) $y_o = 5\frac{1}{4}$ inches.

Figure 10.- Concluded.



(a) $y_o = 5/8$ inch.

Figure 11.- Local values of effective thermal conductance from unfinned test stand.



(b) $y_o = 5\frac{1}{4}$ inches.

Figure 11.- Concluded.

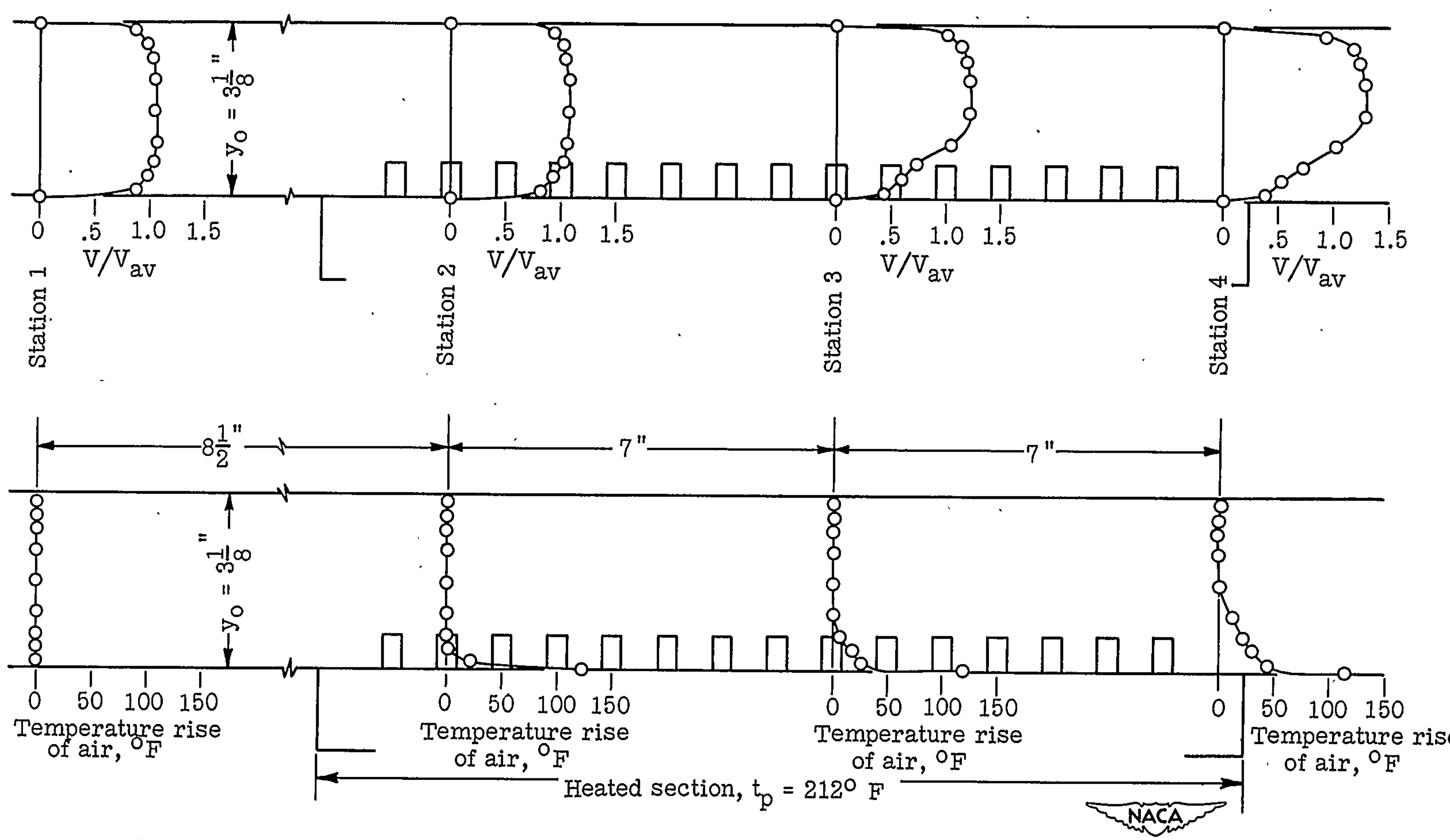


Figure 12.- Isothermal velocity distribution and temperature distribution in duct for steel-pin-fin plate. Average velocity (based on minimum flow area) V_{av}, approximately 30 feet per second.

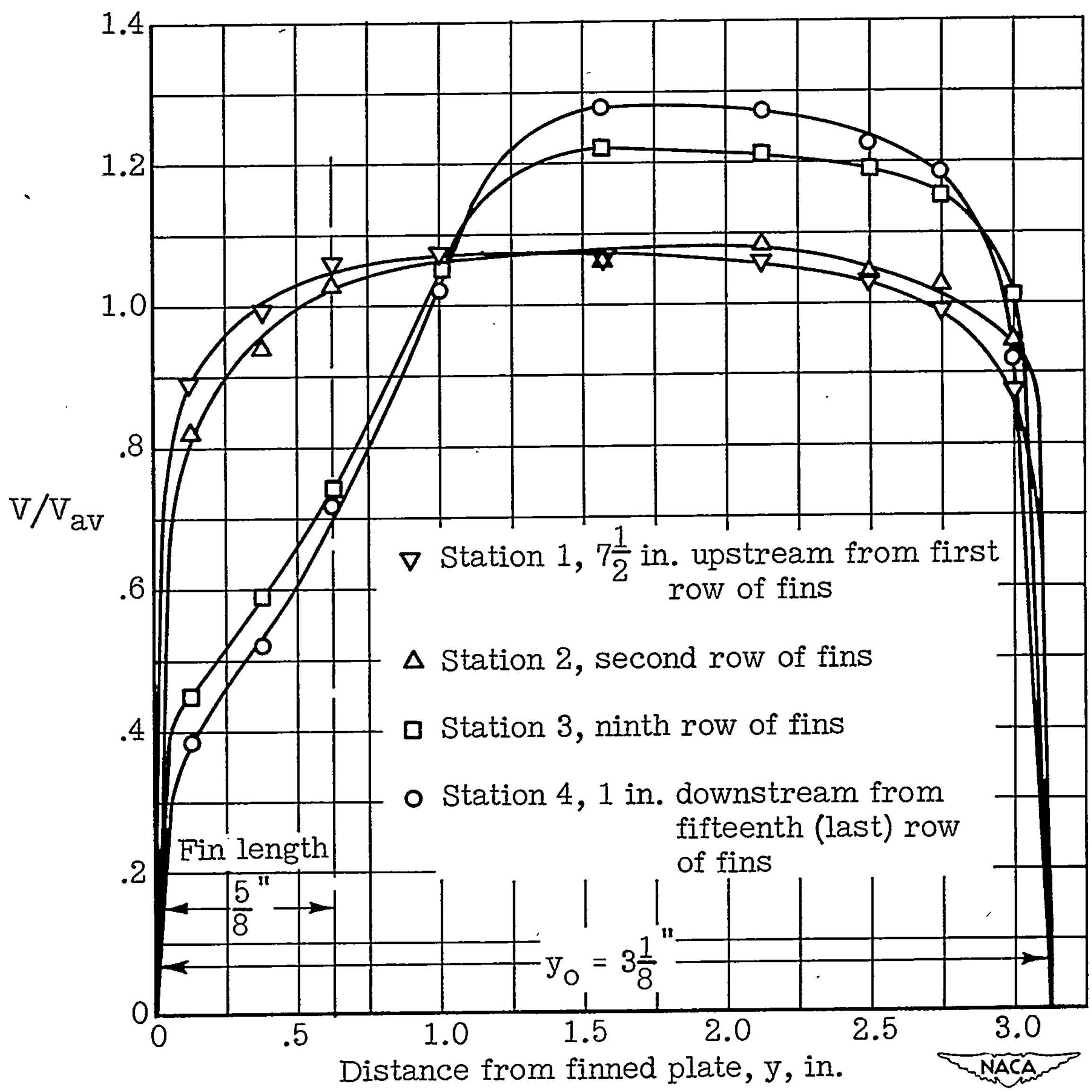
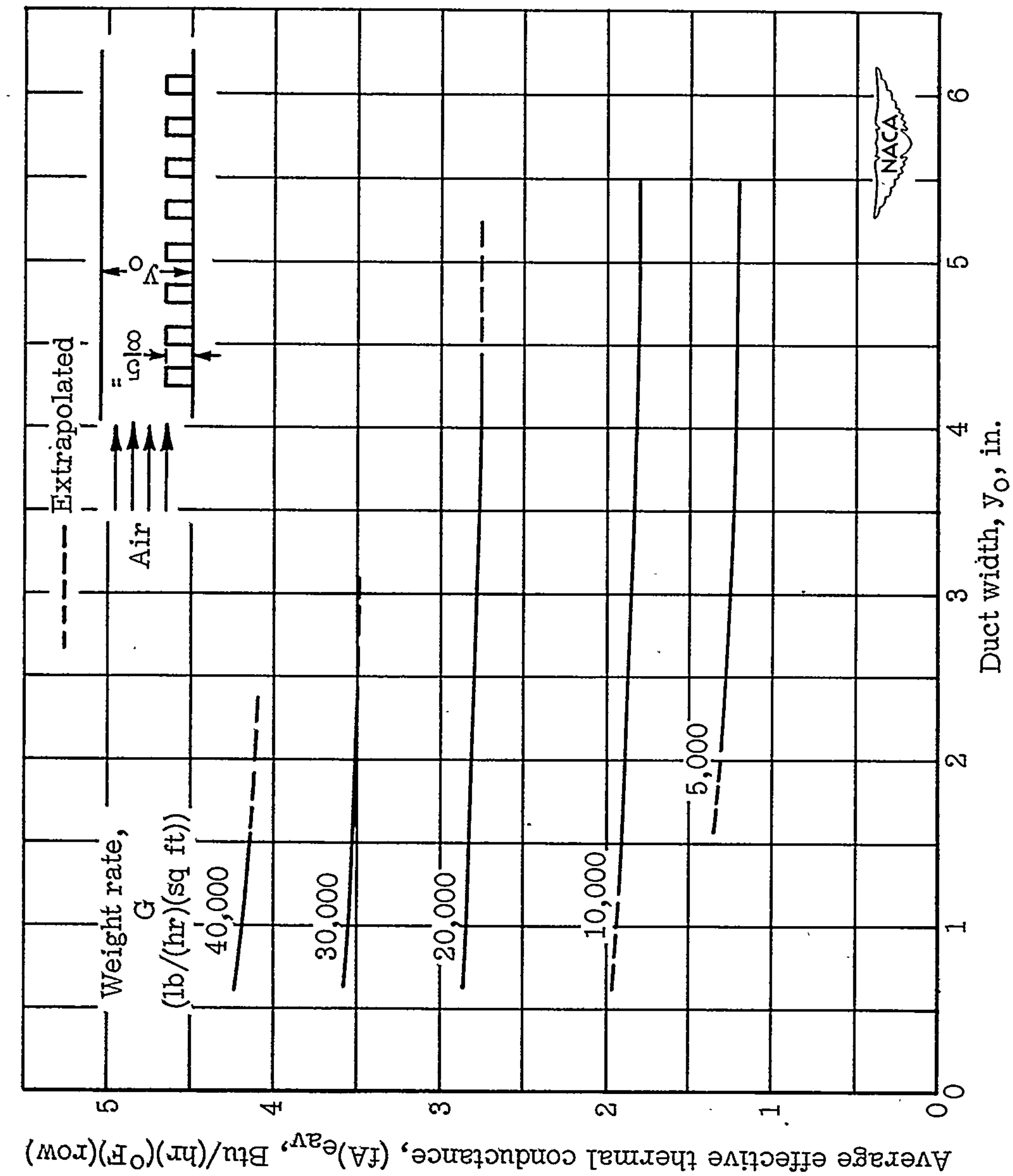
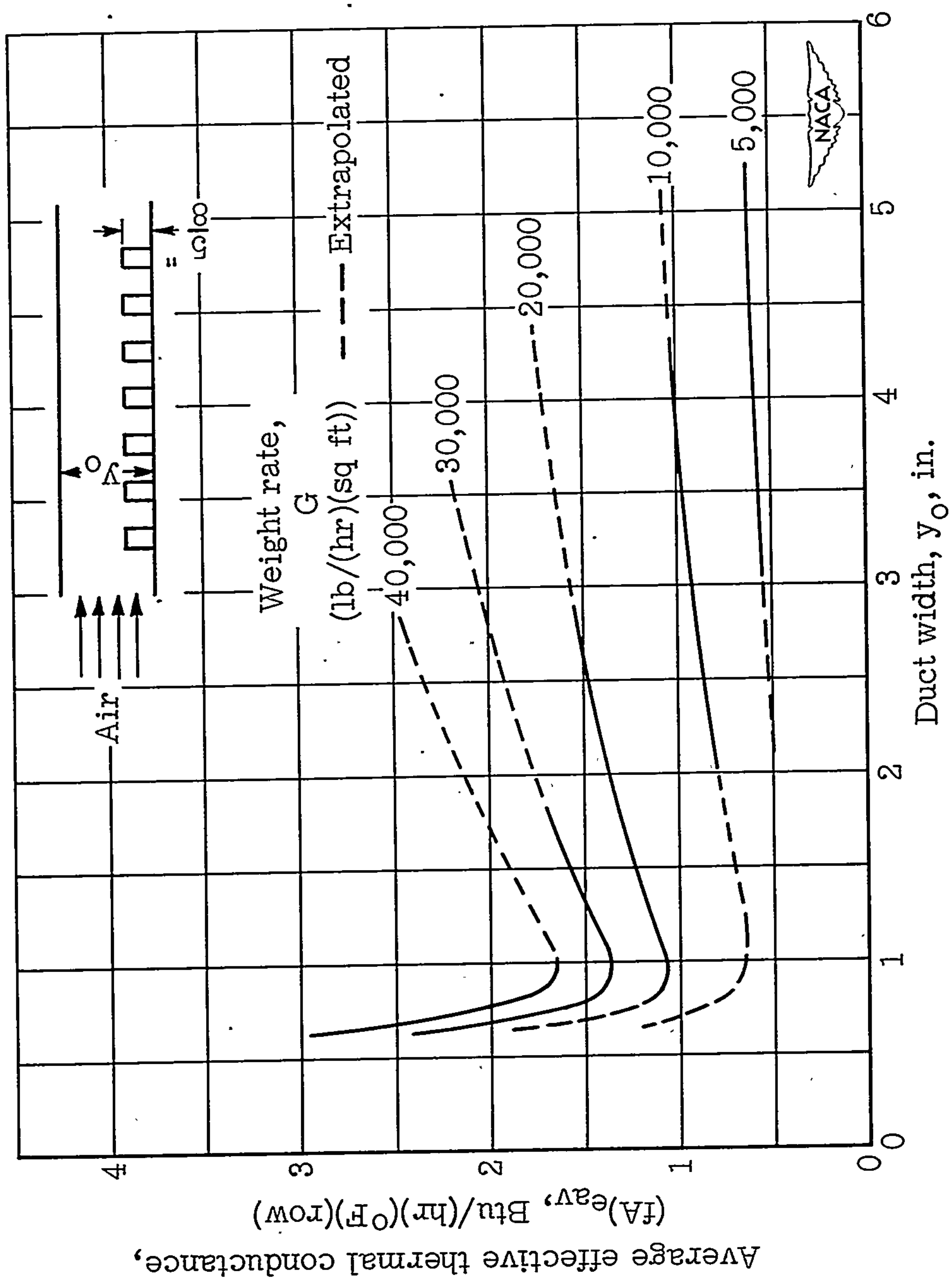


Figure 13.- Isothermal velocity distribution in duct for steel-pin-fin plate. Average velocity (based on minimum flow area) V_{av}, approximately 30 feet per second.



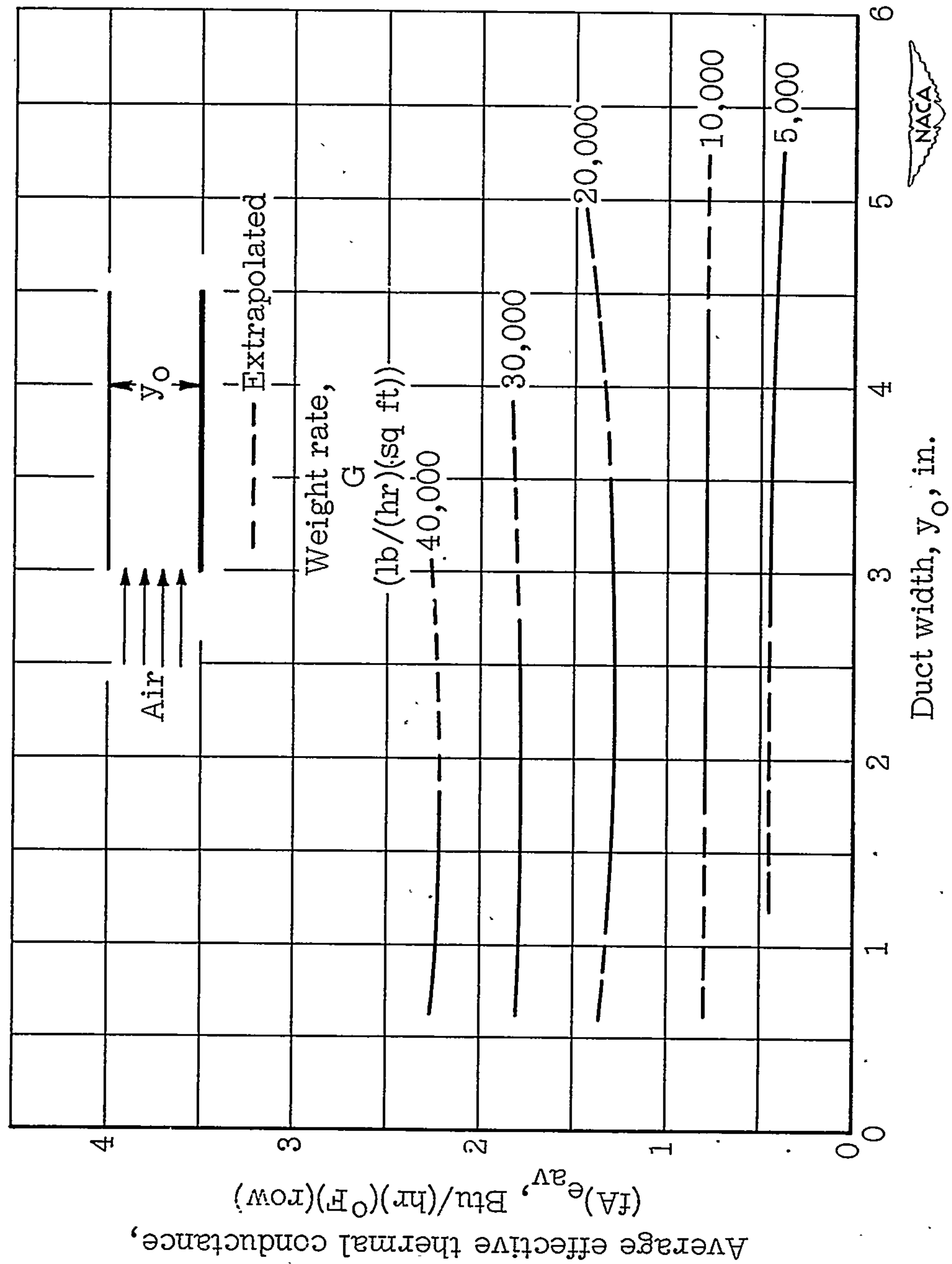
(a) Steel fins.

Figure 14.- Effect on average effective thermal conductance of duct width from pin-fin test stand.



(b) Wooden fins.

Figure 14.- Continued.



(c) Unfinned flat plate.

Figure 14.- Concluded.

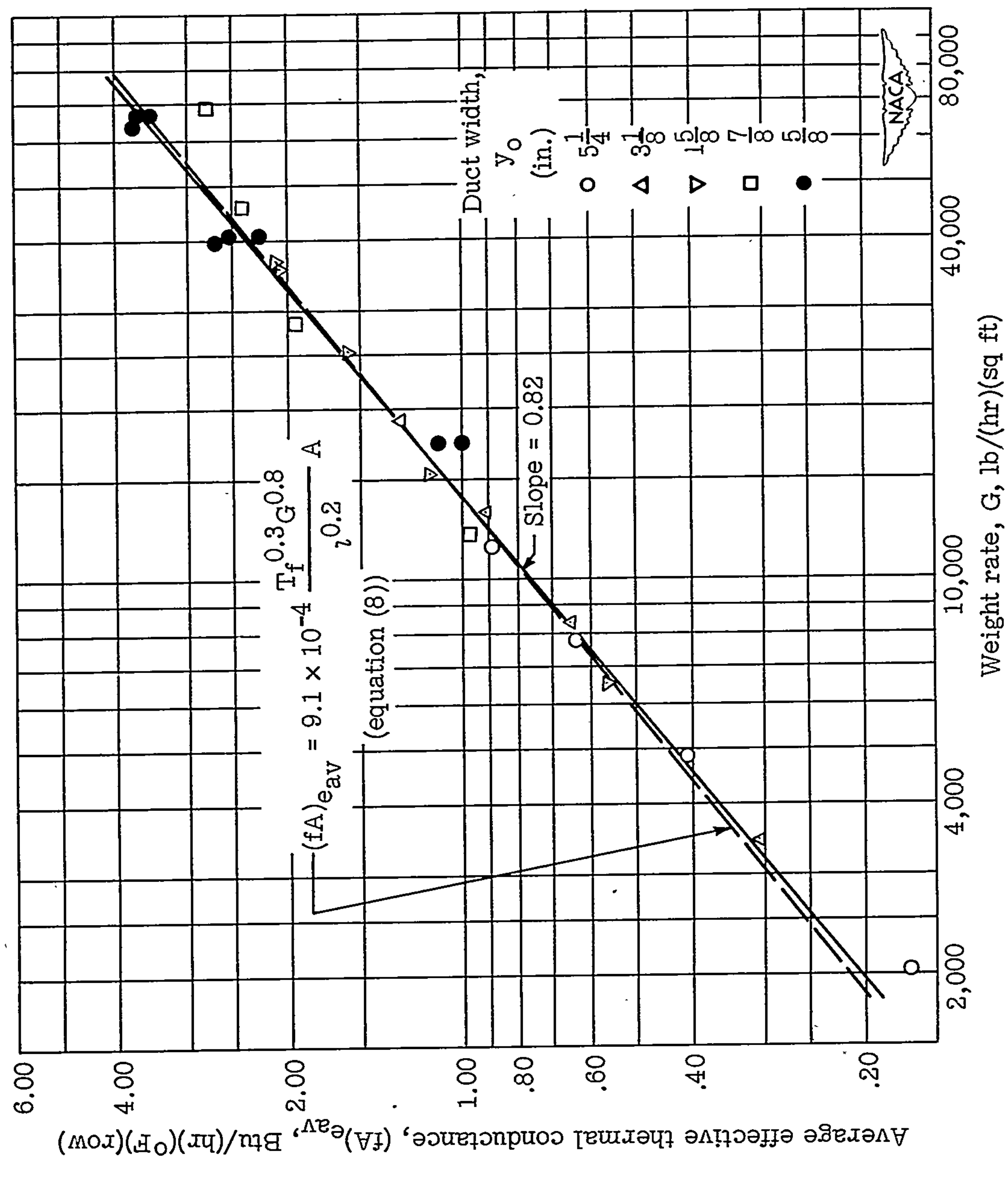
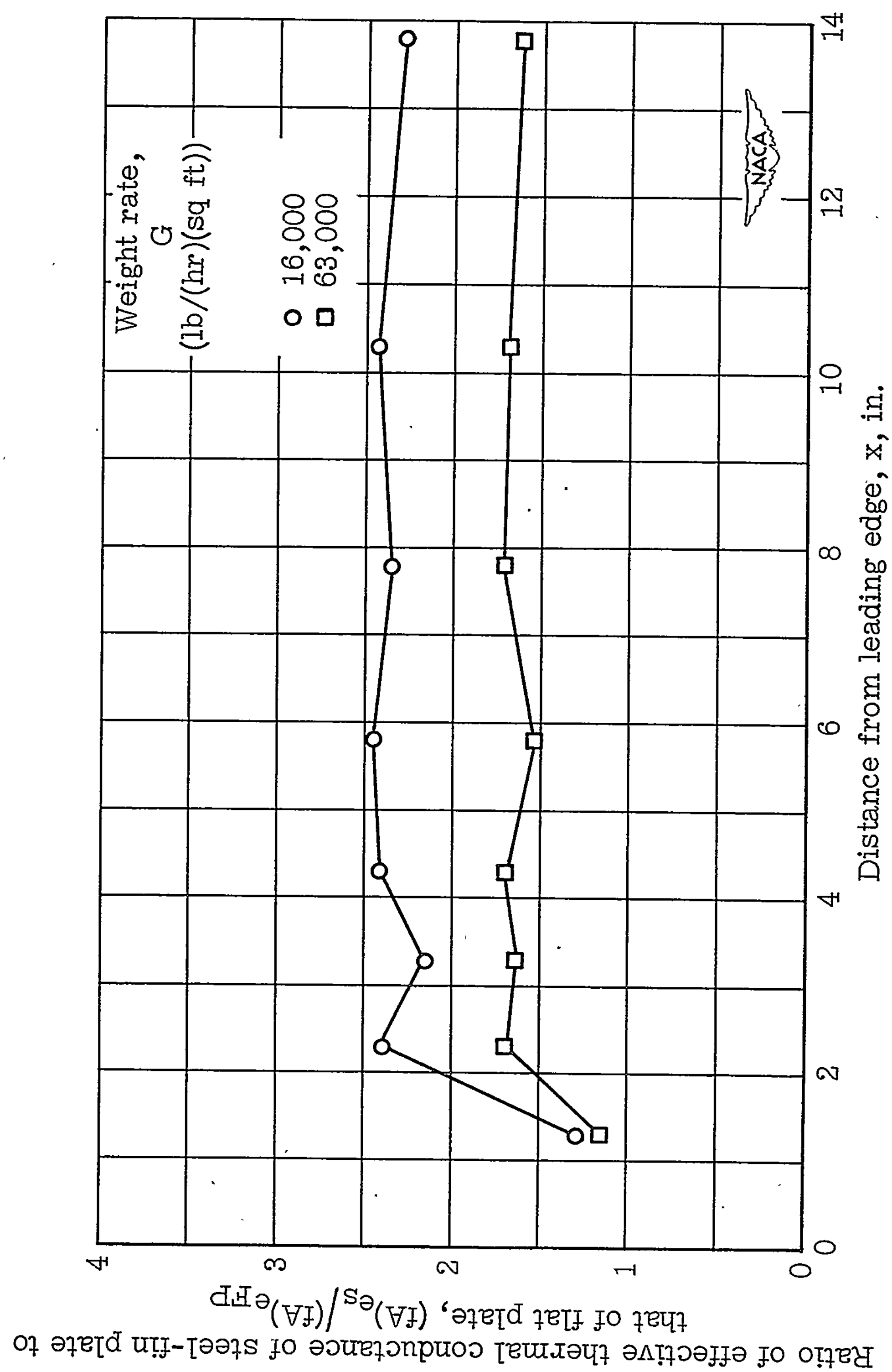
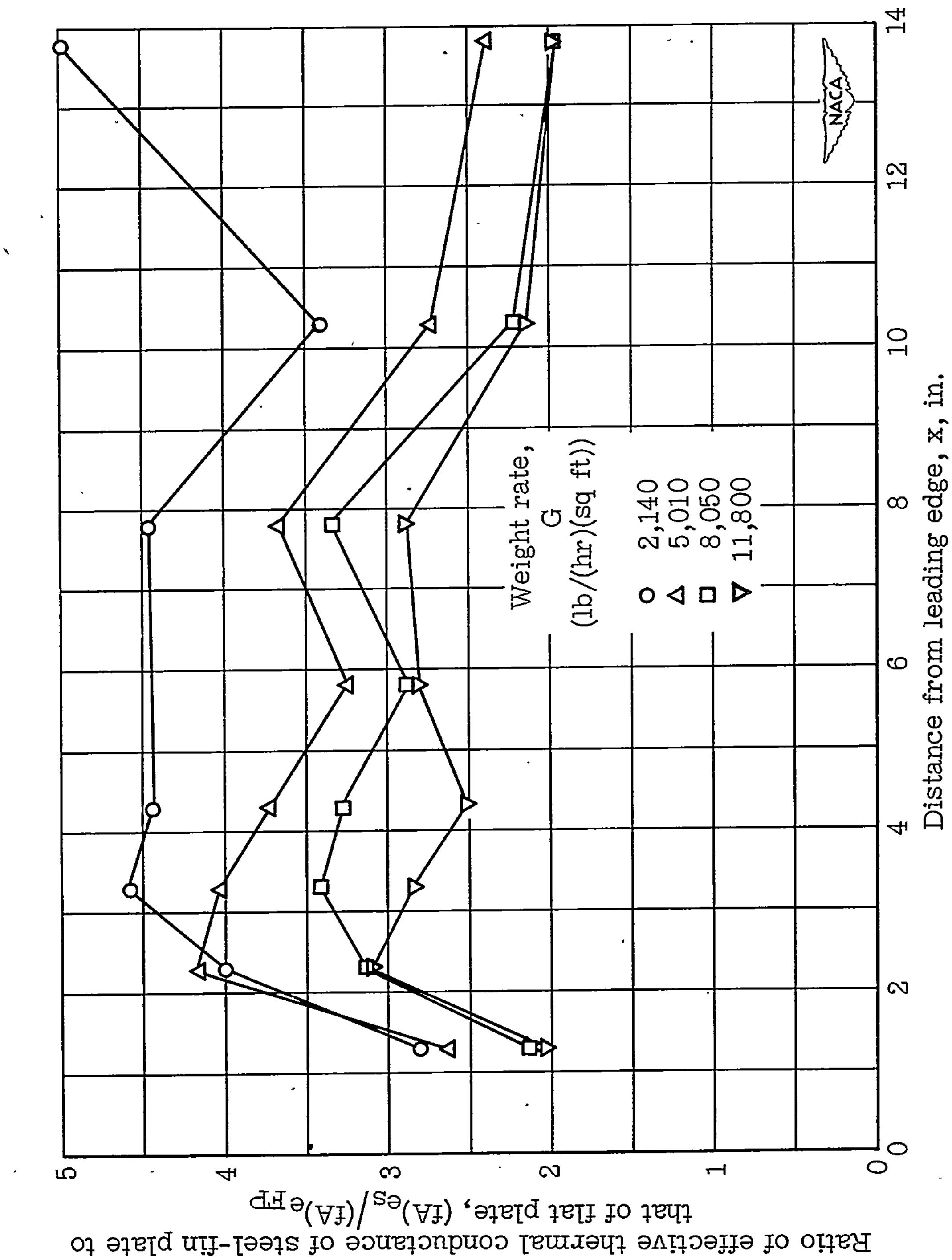


Figure 15.- Variation of average effective thermal conductance for flat-plate unit with weight rate per unit area.



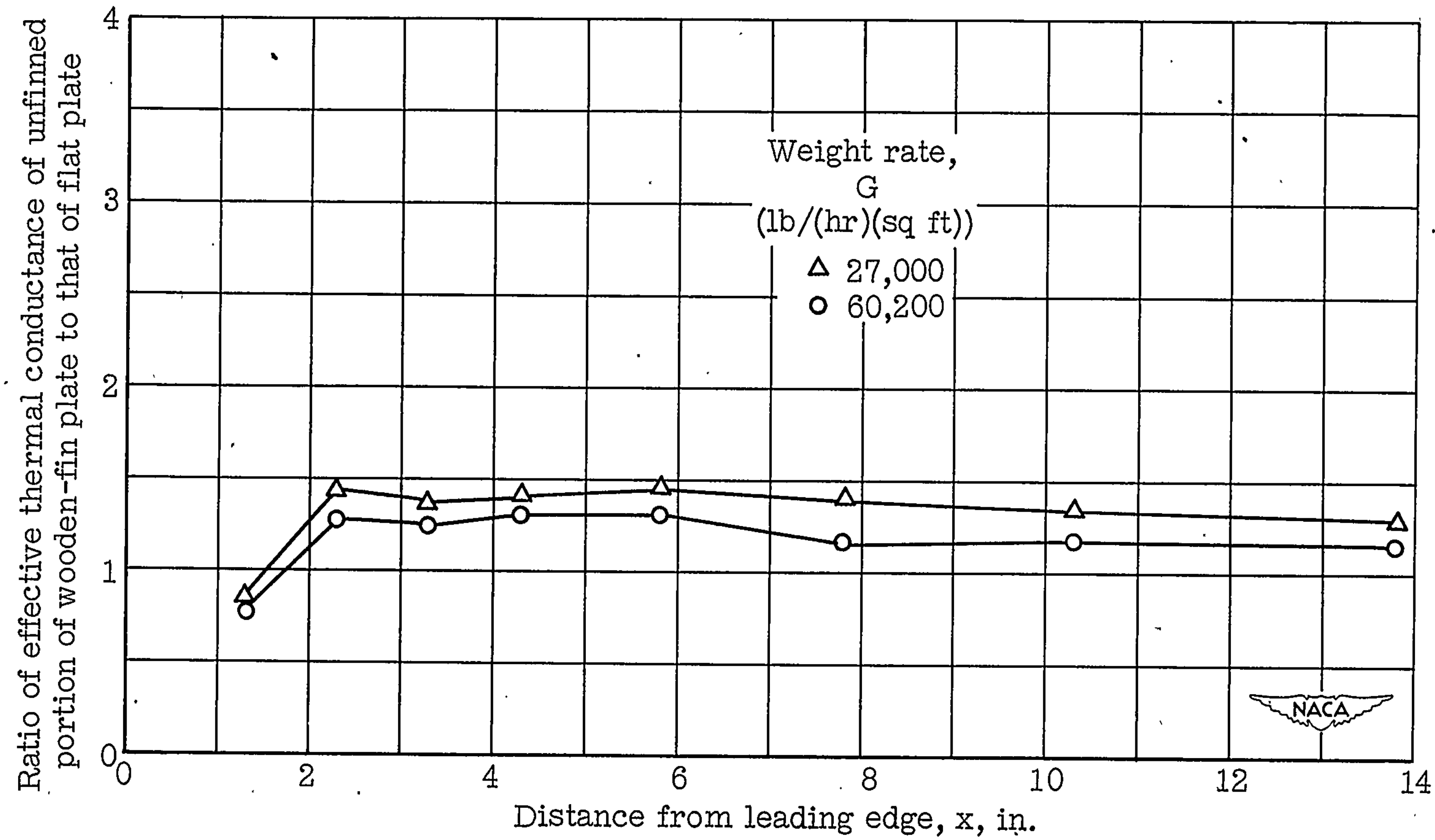
(a) $y_o = 5/8$ inch.

Figure 16.- Over-all effect of finning.



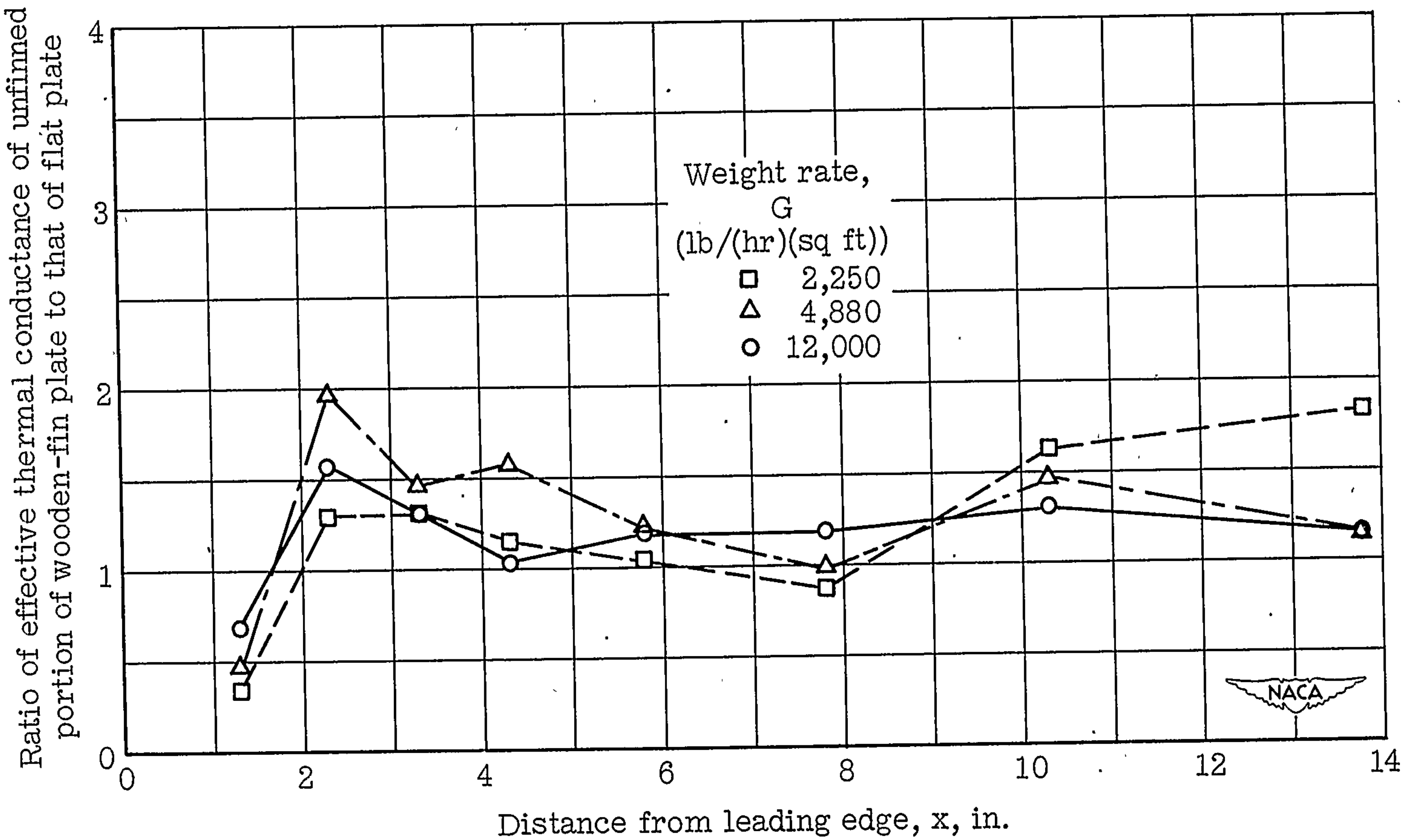
(b) $y_o = 5\frac{1}{4}$ inches.

Figure 16.- Concluded.



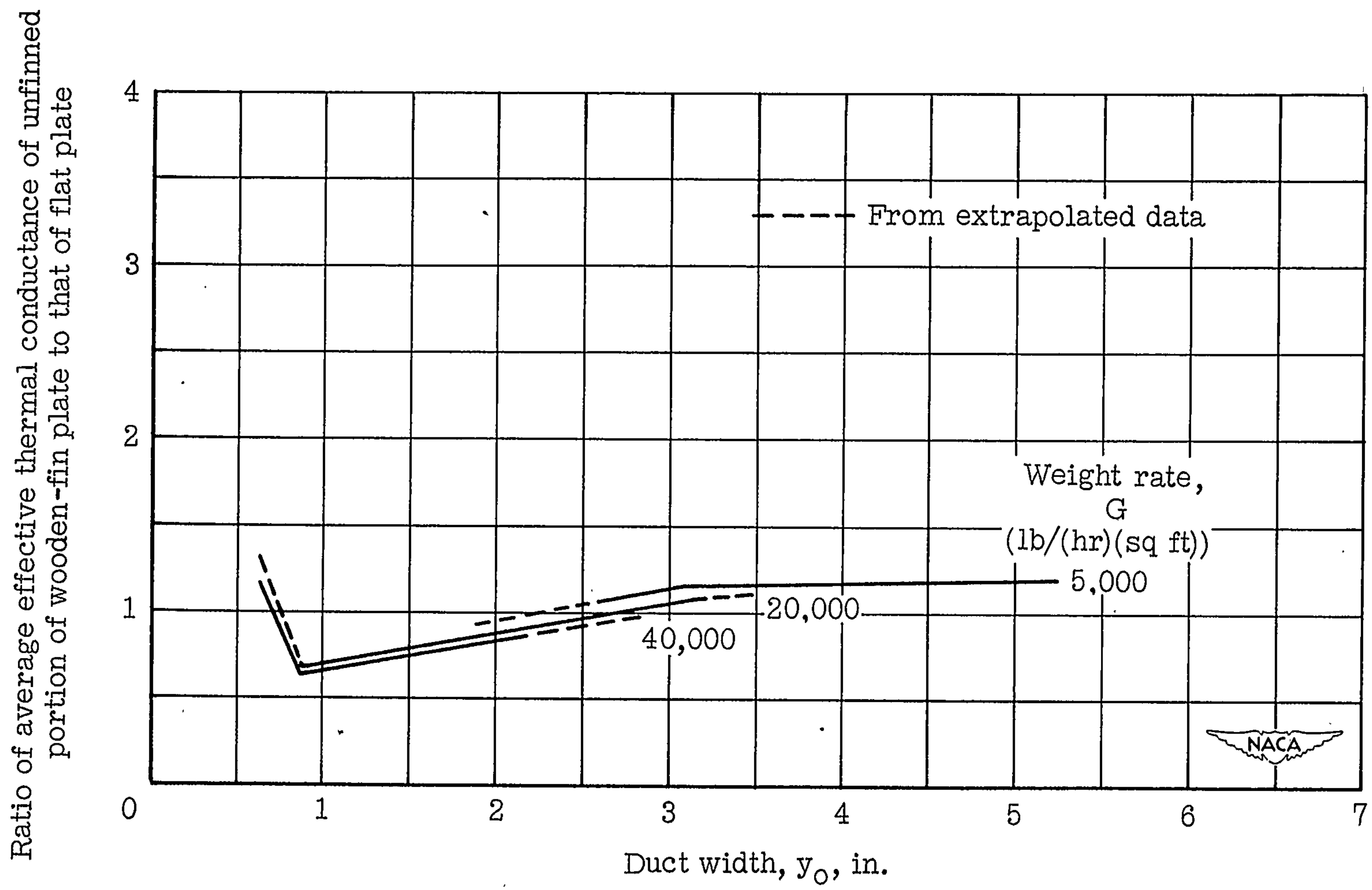
(a) y_o = 5/8 inch.

Figure 17.- Effect of turbulence induced by pin fins on flat plate. (Analytical corrections were made for the portion of heat transferred through the wooden fins; adjustments accounting for the flat-plate area occupied by the wooden fins were also made.)



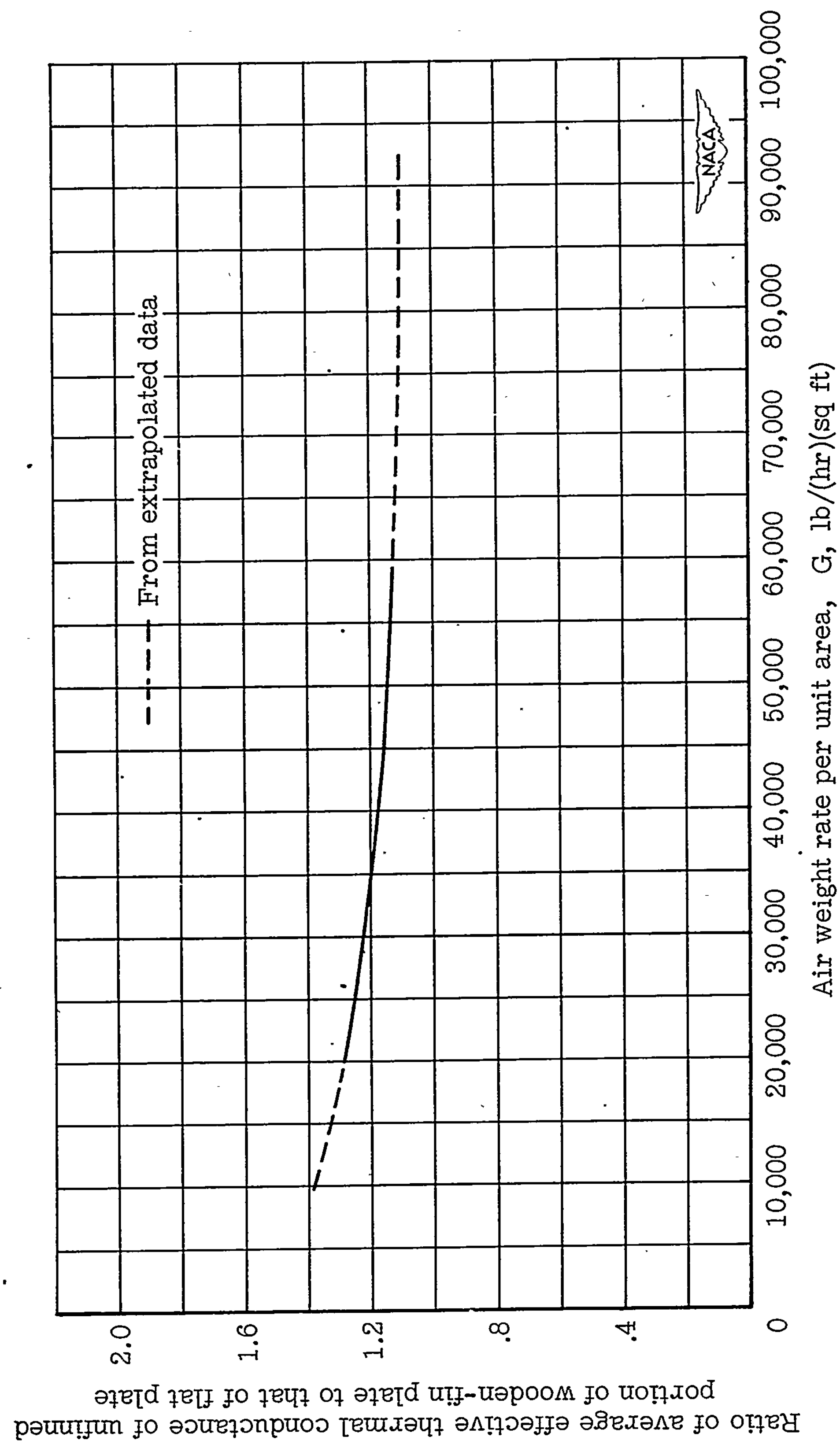
(b) $y_o = 5\frac{1}{4}$ inches.

Figure 17.- Concluded.



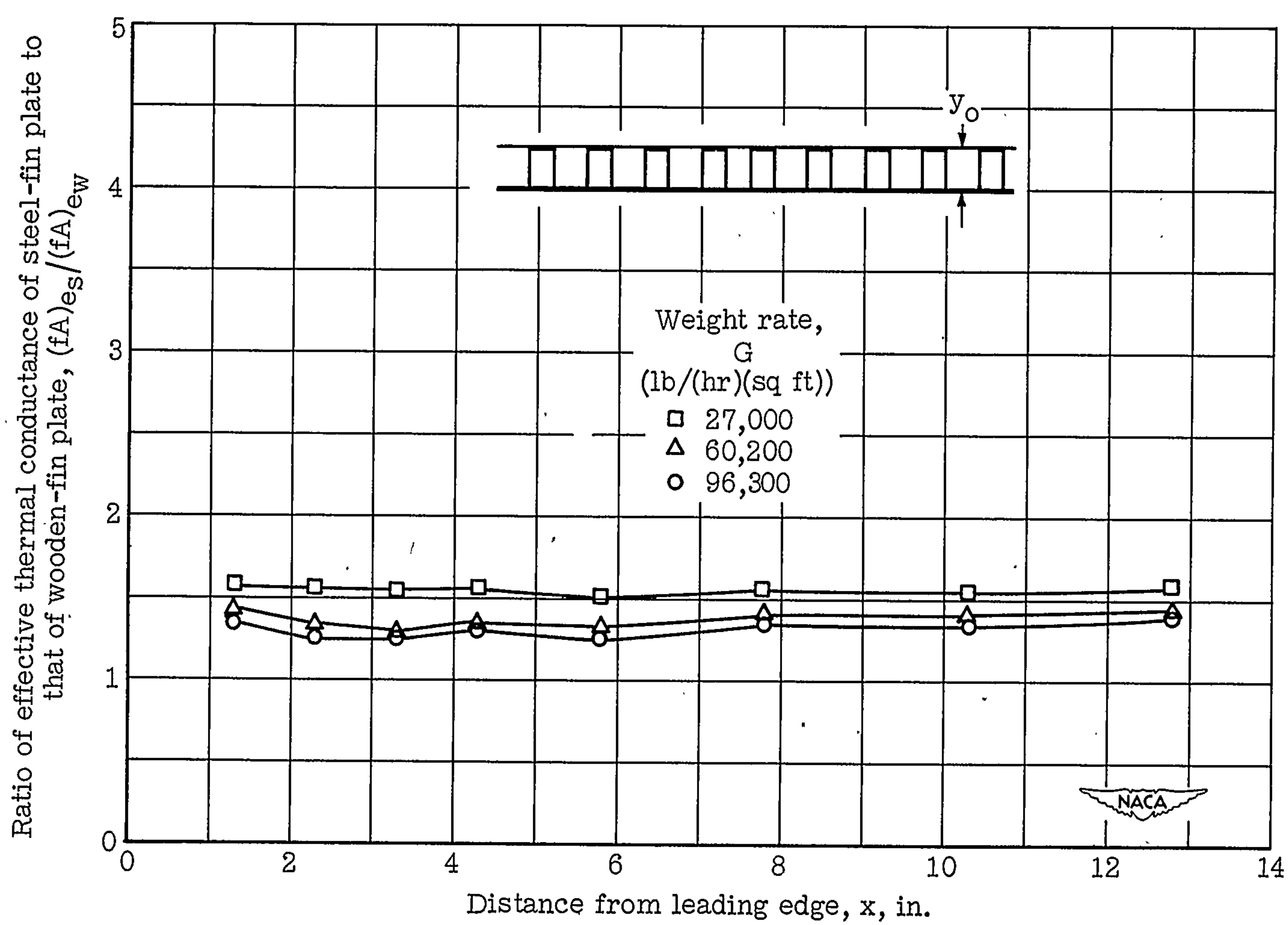
(a) Effect of duct width.

Figure 18.- Average effect of turbulence induced by pin fins on flat plate. (Analytical corrections were made for the portion of heat transferred through the wooden fins; adjustments accounting for the flat-plate area occupied by the wooden fins were also made.)



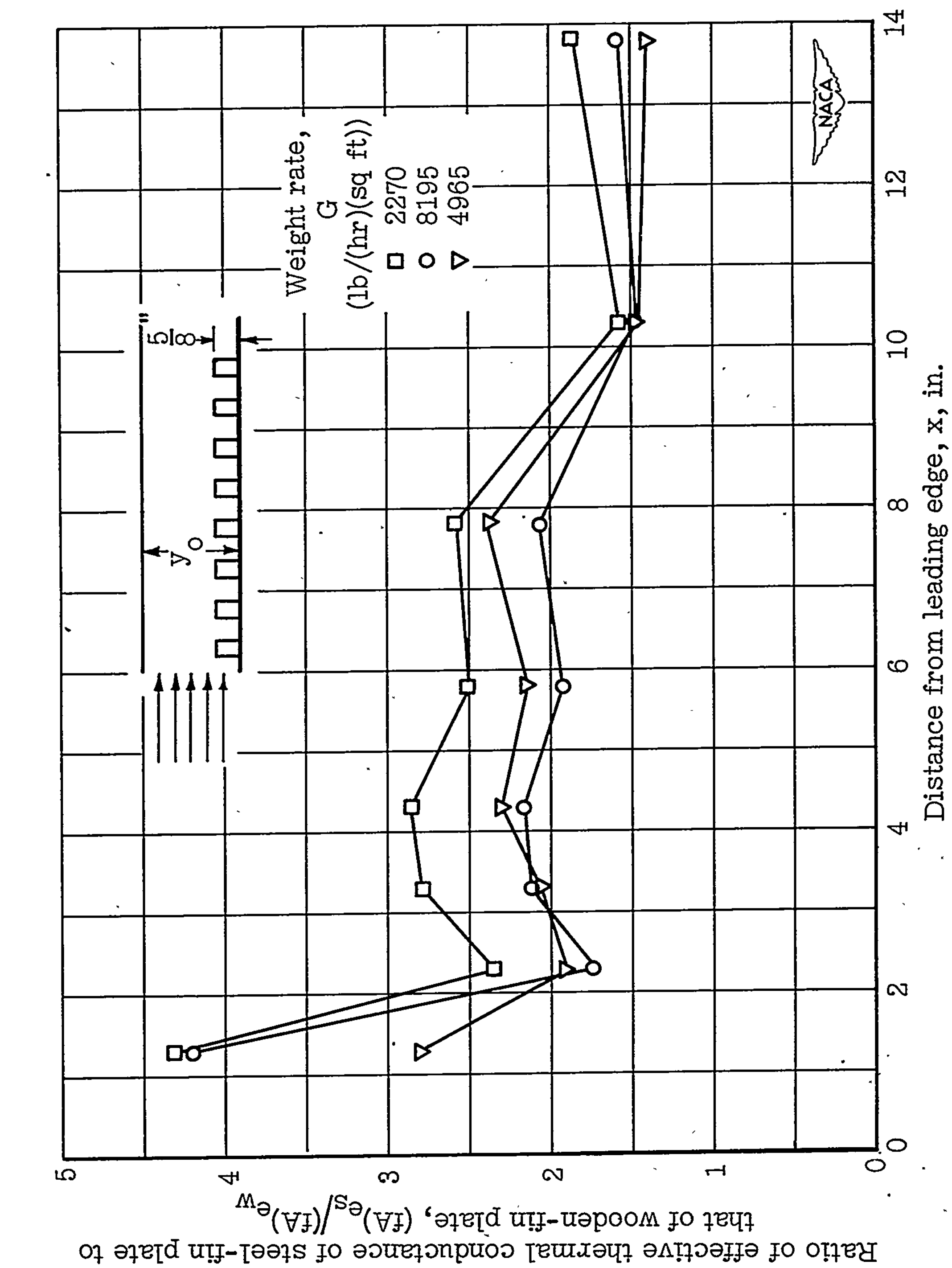
(b) Effect of weight rate; y_o = 5/8 inch.

Figure 18.- Concluded.



(a) y_o = 5/8 inch.

Figure 19.- Effect of thermal conductivity of pin fins on pin-fin plate. (Thermal conductivity of steel k_s, 26 Btu/(hr)(sq ft)($^\circ$F/ft); thermal conductivity of wood k_w, 0.15 Btu/(hr)(sq ft)($^\circ$F/ft).)



(b) $y_o = 5\frac{1}{4}$ inches.

Figure 19.- Concluded.

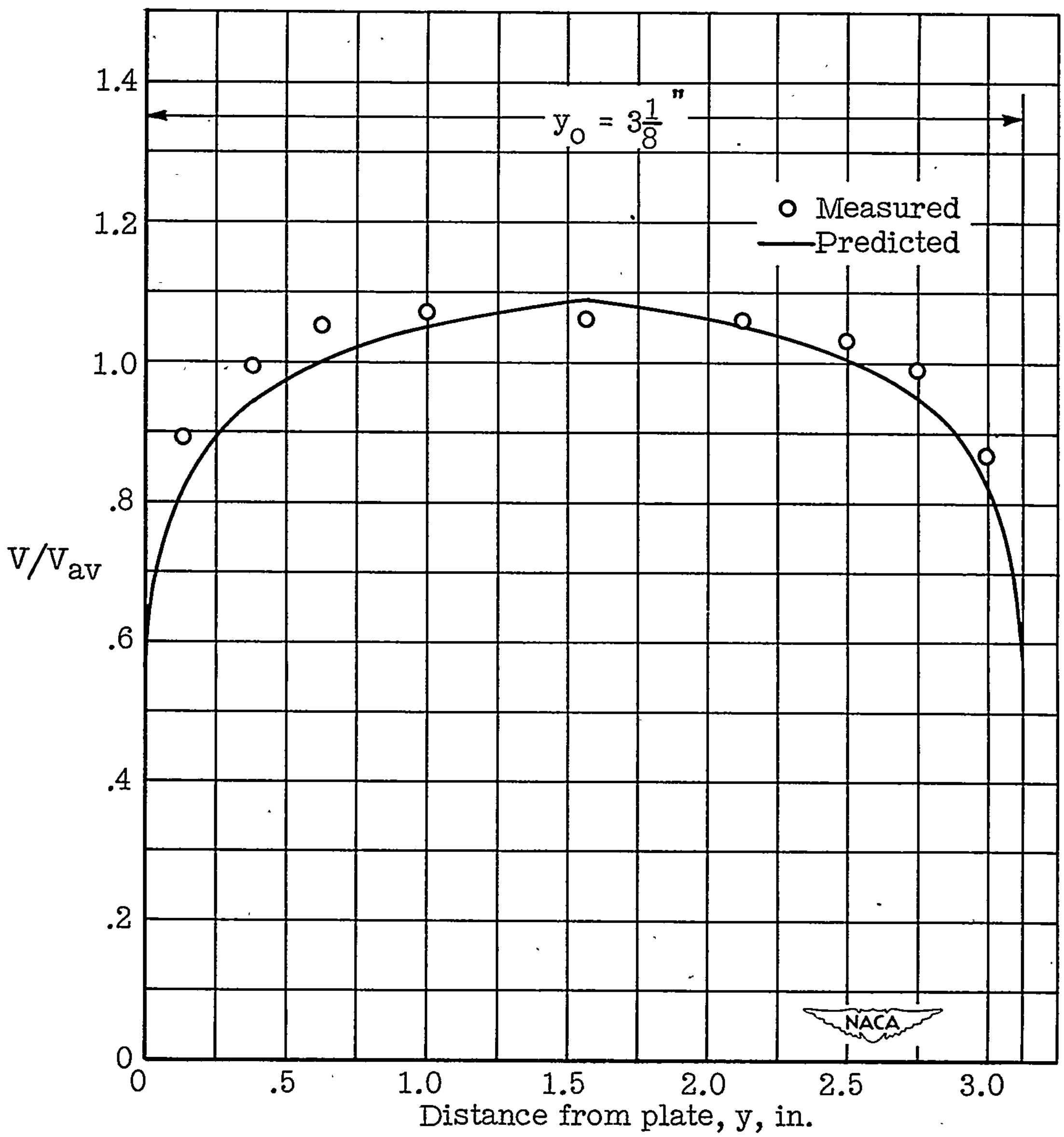


Figure 20.- Comparison of measured and predicted isothermal velocity distribution in duct at station 1 $\left(7\frac{1}{2}\right.$ in. upstream from first row of $\left.\text{fins}\right)$. Reynolds number Re, approximately 100,000; average velocity (based on minimum flow area) V_{av}, approximately 30 feet per second.

$$\frac{V}{V_{av}} = \left(\frac{5.5 + 2.5 \log_e \frac{Re}{2} \frac{y}{1.56} \sqrt{\frac{\zeta}{8}}}{5.5 + 2.5 \log_e \frac{Re}{2} \sqrt{\frac{\zeta}{8}}}\right) \frac{V_{max}}{V_{av}} \text{ (reference 9).}$$

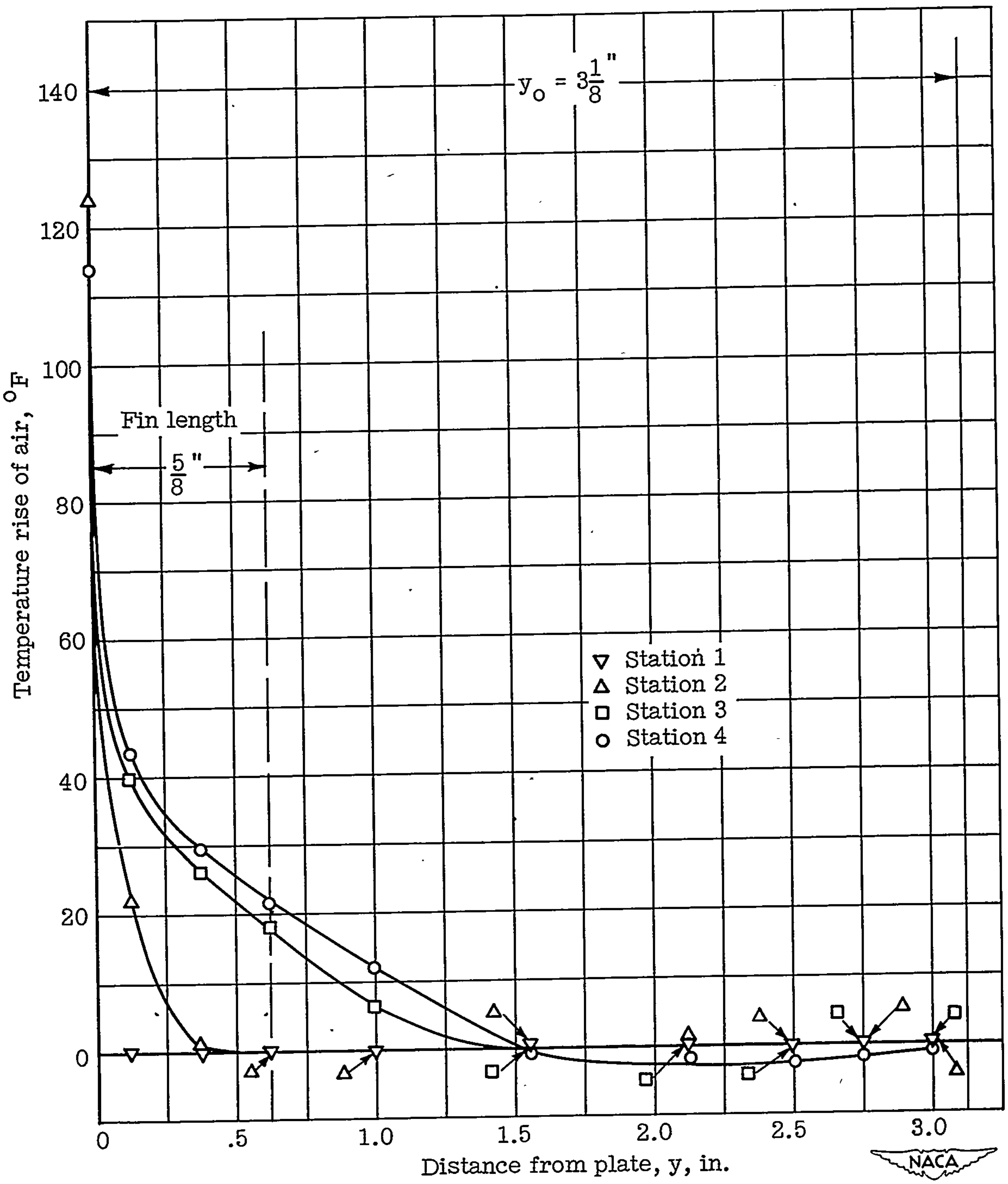


Figure 21.- Temperature distribution in duct for steel-pin-fin plate. Plate temperature t_p, 212° F; average velocity (bas... flow area) V_{av}, approximately 30 feet per second.